Final زيت (+4)

Shadi - Key

BIRZEIT UNIVERSITY

MATH 132 Final exam 09/10

Name (.............................. ....)

INSTRUCTORS : RASEM + SHADI

NO:

PROBLEM 1:10 MARKS

Find the four fourth roots of $-8+8\sqrt{3}\,i$

The modulus $= \sqrt{(-8)^2+(8\sqrt{3})^2} = 16$ (2)

64

$\cos\theta = \frac{-8}{16} = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-8}{16}\right) = 120 = \boxed{\frac{2\pi}{3}}$ (2)

$w_k = 16^{1/4}\left[\cos\left(\frac{120+2k180}{4}\right) + i\sin\left(\frac{120+2k180}{4}\right)\right]$ $k=0,1,2,3$ (2)

$w_0 = 2\,[\cos(30) + i\sin(30)] = 2\left[\frac{\sqrt{3}}{2} + \frac{1}{2}i\right] = \sqrt{3} + i$ (1)

$w_1 = 2\,[\cos(120) + i\sin(120)] = 2\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = -1 + i\sqrt{3}$ (1)

$w_2 = 2\,[\cos(210) + i\sin(210)] = 2\left[-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right] = -\sqrt{3} - i$ (1)

$w_3 = 2\,[\cos 300 + i\sin 300] = 2\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = 1 - i\sqrt{3}$ (1)

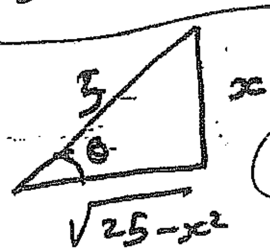
$$\left.\begin{aligned} w_0 &= \sqrt{3} + i \\ w_1 &= -1 + i\sqrt{3} \\ w_2 &= -\sqrt{3} - i \\ w_3 &= 1 - i\sqrt{3} \end{aligned}\right\}$$

PROBLEM 2:10 MARKS EVALUATE

$$\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$$

$x = 5\sin\theta$ (2)

$dx = 5\cos\theta\,d\theta$



(2)

$$\int \frac{5\cos\theta\,d\theta}{25\sin^2\theta\sqrt{25-25\sin^2\theta}}$$ (2)

$$\int \frac{5\cos\theta\,d\theta}{25\sin^2\theta\,(5\cos\theta)}$$ (2)

$$\frac{1}{25}\int \csc^2\theta\,d\theta$$

$$= -\frac{1}{25}\cot(\theta) + C$$

$$= -\frac{1}{25}\frac{\sqrt{25-x^2}}{x} + C$$ (2)

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PROBLEM 3(10 MARKS):COSIDER S IS CONSTANT EVALU.

$$\int_0^{\infty} t^2 e^{-st}\, dt$$

t^2	e^{-st}
$2t$	$-\frac{e^{-st}}{s}$
2	$\frac{e^{-st}}{s^2}$
0	$-\frac{e^{-st}}{s^3}$

$$\lim_{B\to\infty} \int_0^{B} t^2 e^{-st}\, dt$$

$$= \lim_{B\to\infty} \left[-\frac{t^2}{s e^{st}} - \frac{2t}{s^2 e^{st}} - \frac{2}{s^3 e^{st}} \right]_0^{B}$$

$$= \lim_{B\to\infty} \frac{-B^2}{s e^{sB}} - \frac{2B}{s^2 e^{sB}} - \frac{2}{s^3 e^{sB}} - \left(0 - 0 - \frac{2}{s^3}\right)$$

$$= \frac{2}{s^3}$$

PROBLEM 4(10 MARKS)DISCUSS THE CONVERGENCE

$$\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(x+4)^k$$

By Ratio test

$$\lim_{k\to\infty} \left| \frac{\frac{(k+1)^2}{2^{3(k+1)}}(x+4)^{k+1}}{\frac{k^2}{2^{3k}}(x+4)^k} \right| < 1$$

$$= \lim \frac{(k+1)^2}{k^2}\left(\frac{1}{2^3}\right)|x+4| < 1$$

$$= \lim \frac{1}{8}|x+4| < 1$$

$$= |x+4| < 8 \longrightarrow \boxed{R=8}$$

$$-8 < x+4 < 8$$

$$\boxed{-12 < x < 4}$$

when $\boxed{x=-12}$ $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(-12+4)^k = \sum \frac{k^2}{(8)^k}(-8)^k = \sum_{k=0}^{\infty}(-1)^k k^2$

$\lim_{k\to\infty} k^2 = \infty$ div.

when $\boxed{x=4}$ $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(4+4)^k = \sum_{k=0}^{\infty} k^2$

$\lim k^2 = \infty$ div

The interval of conv. $(-12, 4)$

PROBLEM 5: 60 MARKS

Consider the test form the first column and the result form the second column

1. geometric series
2. p- series
3. telescoping series
4. the nth-term test
5. the integral test
6. alternating series test
7. the direct comparison test
8. the limit comparison test
9. the ratio test
10. the root test

a. converges absolutely
b. converges conditionally
c. diverges

solve in details then circle the correct answer

1. radius of conv.

$$\sum_{k=1}^{\infty}\frac{2^k(x-3)^k}{\sqrt{k+3}} =$$

a. $\frac{1}{2}$

b. $\frac{7}{2}$

c. $\frac{19}{6}$

d. $\frac{5}{2}$

$\text{Lim}\left|\frac{2^{k+1}(x-3)^{k+1}}{\sqrt{k+4}} \cdot \frac{\sqrt{k+3}}{2^k(x-3)^k}\right| < 1$

$= \text{Lim } 2|x-3| \frac{\sqrt{k+3}}{\sqrt{k+4}} < 1$

$= 2|x-3| < 1$

$|x-3| < \frac{1}{2}$

$-\frac{1}{2} < x-3 < \frac{1}{2}$

$\frac{5}{2} < x < \frac{7}{2}$

$x = \frac{5}{2}$ by conv. by alternating

$x = \frac{7}{2}$ div. by limit cmp. test

2. $\sum_{k=1}^{\infty}\frac{\sin k}{k^2+1}$

a. divergent by p- series
b. divergent by geometric series
c. converges conditionally
d. converges absolutely

$a_k = \left|\frac{\sin k}{k^2+1}\right| \le \frac{1}{k^2+1} \le \frac{1}{k^2}$

$\sum \frac{1}{k^2}$ conv. p-se

$\sum a_k$ conv. by D.C.T.

3. $\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{\sqrt{k+1}}\right) =$

a. divergent by the ratio test
b. divergent by the nth-root test
c. converges conditionally
d. converges absolutely

1) Let $a_k = \frac{1}{\sqrt{k+1}} > 0$

2) $f(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$

$f'(x) = \frac{-1}{2\sqrt{(x+1)^3}} < 0$

decreasing series

3) $\text{Lim } a_k = 0$

$\sum (-1)^k a_k$ conv. alternating series

Let $b_k = \frac{1}{\sqrt{k}} = \frac{1}{k^{1/2}}$

$\lim \frac{a_k}{b_k} = \lim \frac{\sqrt{k}}{\sqrt{k+1}} = 1$

$\sum b_k$ div. p-series

$\Rightarrow \sum a_k$ div. by Limit C.T.

4. $\sum_{k=2}^{\infty}\frac{(-1)^k\sqrt{k}}{\ln k}=$

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

$\lim \frac{\sqrt{k}}{\ln k} = \lim \frac{\frac{1}{2\sqrt{k}}}{1/k} = \lim \frac{\sqrt{k}}{2} \neq 0$

By k-th term test

the series div.

5. $\sum_{k=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}=$

a. convergent by the integral test
b divergent .by limit comparison test
c. convergent by limit comparison test
d. divergent by by direct comparison test

Let $a_k = \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

$b_n = \frac{1}{n^2}$

$\lim \frac{a_n}{b_n} = \frac{\sqrt{n^2-1}\; n^2}{n^3+2n^2+5} =$

But $\sum b_n = \sum \frac{1}{n^2}$

Conv. p-series

So $\sum a_n$ Conv. Limit C.T.

6. $\sum_{k=0}^{\infty}\frac{(-2)^{3k-1}}{9^k}= \sum_{k=0} \frac{(-2)^{-1}\left((-2)^3\right)^k}{9^k} = \sum_{k=0} -\frac{1}{2}\left(\frac{-8}{9}\right)^k$

a. $\frac{-2}{9}$

b. $\frac{9}{7}$

c. $\frac{-9}{7}$

d. $\frac{-9}{34}$

$= -\frac{1}{2}\left[\frac{1}{1-(\frac{-8}{9})}\right]$

$= -\frac{1}{2}\left[\frac{1}{1+\frac{8}{9}}\right]$

$= -\frac{1}{2}\left[\frac{9}{9+8}\right] = \frac{-9}{34}$

7. $\sum_{k=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}=$

a. convergent by the integral test
b divergent .by limit comparison test
c. convergent by telescoping series
d. divergent by by direct comparison test

$\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx$

$u = \frac{1}{x}$, $du = -\frac{1}{x^2}dx$ | $x=1 \to u=1$, $x \to \infty \to u \to 0$

$-\int_1^0 e^u\,du = \cancel{e^u\Big]}$

$= \int_0^1 e^u\,du = e^u\Big]_0^1$

$= e - 1$

$e^{1/x}, \frac{1}{x^2}$ Cont., +ve, dec.

8. $\sum_{k=1}^{\infty} \frac{\cos(\frac{k\pi}{6})}{k\sqrt{k}} =$

$a_k \le \frac{|\cos k\pi|}{k\sqrt{k}} \le \frac{1}{k^{3/2}} = b_k$

Σb_k conv. p-series $\Rightarrow \Sigma a_k$ con by D.C.T.

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

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9. $\lim_{x\to 0}(\cos 3x)^{\frac{5}{x}} =$

a. 1
b. 0
c. $\frac{1}{2}$
d. ∞

Let $y = (\cos 3x)^{5/x} \Rightarrow \ln y = \frac{5\ln(\cos 3x)}{x}$

$\cancel{\ln y = 5\cos 3x}$

$\lim_{x\to 0} \ln y = \lim 5\frac{\ln\cos 3x}{x} = \lim \frac{-15\tan 3x}{1} = 0$

$\lim y = \lim e^{\ln y} = e^0 = 1$

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10. $\lim_{x\to 0}(1-2x)^{\frac{1}{x}} =$

a. does not exit
b. e^{e-2}
c. e^{-2}
d. $\frac{2}{e}$

Let $y = (1-2x)^{\frac{1}{x}}$

$\ln y = \frac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \frac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \frac{-2}{1-2x} = -2$

$\Rightarrow \lim (1-2x)^{\frac{1}{x}} = \lim e^{y} = e^{-2}$

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11. the sequence $a_n = \frac{\ln(2+e^n)}{3n}$

a. converges to $\frac{2}{3}$
b. converges to $\frac{1}{3}$
c. converges to 0
d. divergent sequences

$\lim \frac{\ln(2+e^n)}{3n}$

$\lim \frac{\frac{e^n}{2+e^n}}{3} = \lim \frac{e^n}{6+3e^n}$

$= \lim \frac{e^n}{3e^n} = \frac{1}{3}$

12. $\int \frac{2x^2-3x+2}{x^3+x}$ CAN BE INTERATED BY PARTIAL FRACTION

a. $\frac{A}{X}+\frac{BX+C}{X^2+1}$

b. $\frac{A}{X}+\frac{B}{X^2+1}$

c. $\frac{A}{X}+\frac{C}{X^2+1}+\frac{D}{X^2}$

d. $\frac{A}{X}+\frac{BX^2+C}{X^3}$

13. $\int_0^1 \tan^{-1}(x)dx$

$u=\tan^{-1}x$ $\qquad dv = dx$

$du=\frac{1}{1+x^2}dx$ $\qquad v = x$

$x\tan^{-1}x - \int \frac{x}{1+x^2}$

$x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\Big]_0^1$

$\tan^{-1}1 - \tan^{-1}(0) - \left[\frac{1}{2}\ln 2 - \frac{1}{2}\ln 1\right]$

$= \pi/4 - \frac{1}{2}\ln 2$

a. 1
b. 0
c. $\frac{\pi}{4}$
d. $\frac{\pi}{4}-\frac{1}{2}\ln 2$

14. $\int_1^{e^{\frac{\pi}{2}}} \frac{\cos(\ln x)}{x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$\int_0^{\pi/2}\cos u\,du = \sin u\Big]_0^{\pi/2} = 1$

a. diverge
b. 1
c. 0
d. e

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15. $\int_0^1 \frac{x}{1+3x}dx$

a. $\frac{1}{3}-\frac{1}{3}\ln 4 = \frac{1}{3}\int_0^1 \frac{3(x)+1}{1+3x}dx - \int_0^1 \frac{1}{1+3x}dx$

b. $\frac{1}{3}$

c. $\frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3}$

$= \frac{1}{3}x\Big]_0^1 - \frac{1}{3}\ln(1+3x)\Big]_0^1$

d. 1/4 $\frac{1}{3}+\frac{1}{3}\ln 2$

$= \frac{1}{3} - \frac{1}{3}\ln 4$

16. $y = e^{\sinh x}$

a. $y' = \cosh x$

b. $y' = \cosh x\, e^{\sinh x}$

c. $y' = \sinh x\, e^{\sinh x}$

d. $y' = e^{\cosh x}$

17. **the center of the ellipse**

$4x^2 + y^2 - 8x + 4y - 8 = 0$

Is

a.(2,4)

b. (4,2)

c.(1,2)

d. (1,-2)

$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$

$4(x-1)^2 + (y+2)^2 = 16$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

18.**the equation of the asymptotes for the hyperbola**

$4x^2 - 3y^2 + 8x + 16 = 0$

a. $y = \frac{2}{\sqrt{3}}(x+1)$ **and** $y = -\frac{2}{\sqrt{3}}(x+1)$

b. $y = \frac{2}{\sqrt{3}}(x)$ **and** $y = -\frac{2}{\sqrt{3}}(x)$

c. $y - 1 = \frac{2}{\sqrt{3}}(x)$ **and** $y - 1 = -\frac{2}{\sqrt{3}}(x)$

d. $y = (x-1)$ **and** $y = -(x-1)$

$\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$

$a = 2$

$b = \sqrt{3}$

19. **the focus of the parabola** $2y = 1 - 2x - x^2$ **is**

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$(x+1)^2 = -2(y-1)$

$(x-h)^2 = 4p(y-k)$

$(h,k) = (-1,1)$

$p = -\frac{1}{2}$

Focus $= (-1, \frac{1}{2})$

19) the focus of the parabola $2y = 1 - x - x^2$ is.

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$x^2 + x - 1 = -2y$

$x^2 + x + \frac{1}{4} = -2y + 1 + \frac{1}{4}$

$(x + \frac{1}{2})^2 = -2(y - \frac{5}{8})$

20. Consider the function $f(x) = \frac{1}{x}$ find the maximum error in using a Taylor polynomial Of order 3 centered at a=1to estimate 1.2

a. $(0.2)^{4}$

b. $(1.2)^3$

c. 1

d. $(\frac{1}{1.2})^3$

$R_3 = \frac{24(x-1)^4}{4!\, c^5}$ $\quad 1 \le c \le 1.2$

$|R_3| \le (1-1.2)^4 = (.2)^4$

21 From quastion20the infinite series represent

$$f(x) = \frac{1}{x} = \sum_{n=0}^{\infty} (1 - x)^n$$

a. true

b. false

$$f(x) = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

Birzeit University

Mathematics Department

Math 132

Final Exam

First Summer Semester 2012/2013

Student Name: ....

Student Number: ....

Time: 150 minutes

There are 4 questions in 10 pages

Question 1. (60%) Circle the most correct answer:

(1) The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$, and the x-axis, about the y-axis, is:

(a) $\frac{3\pi}{5}$

(b) $\frac{\pi}{5}$

(c) $\frac{2\pi}{5}$

(d) $\frac{4\pi}{5}$

(2) $\sum_{n=2}^{\infty}(0.5)^{-n} =$

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) None of the above

(3) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

(a) 0

(b) -1

(c) ∞

(d) $-\infty$

(4) If y is the solution of the differential equation $\frac{dy}{dx} = 3x^2y + y$, $y(1) = e$, then $y(-1) =$

(a) -1

(b) -3

(c) e^{-1}

(d) e^{-3}

(5) $\displaystyle\int_1^4 \frac{3^{\sqrt{x}}}{2\sqrt{x}}\,dx =$

(a) $\dfrac{6}{\ln 3}$

(b) $\dfrac{3}{\ln 3}$

(c) $\dfrac{78}{\ln 3}$

(d) $\dfrac{9}{\ln 3}$

(6) The volume of the solid whose base is the region enclosed between the curves $y = x^2$ and $y = x$, and whose cross sections perpendicular to the x-axis are equilateral triangles of height 4, is:

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{3}$

(c) $\dfrac{1}{6}$

(d) $\dfrac{1}{4}$

(7) If $a_n = n3^{\frac{1}{n}}$, $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} a_n =$

(a) 1

(b) 0

(c) ∞

(d) $\ln 3$

(8) $\displaystyle\sum_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2} =$

(a) -1

(b) 1

(c) $\dfrac{1}{4}$

(d) 2

(9) Assuming its convergence, find the limit of the following recursively defined sequence, $a_1 = 8$, $a_{n+1} = \sqrt{a_n + 8} - 2$:

(a) 1

(b) -4

(c) -2

(d) 8

(10) $\int e^{\sqrt{2x+1}}\,dx =$

(a) $2\sqrt{2x+1}\,e^{\sqrt{2x+1}} + C$

(b) $\dfrac{e^{\sqrt{2x+1}}}{2\sqrt{2x+1}} + C$

(c) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$

(d) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - \sqrt{2x+1} + C.$

(11) If $\tanh x = \dfrac{1}{2}$, $x < 0$, then $\operatorname{sech} x =$

(a) $\dfrac{\sqrt{5}}{2}$

(b) $\dfrac{-\sqrt{5}}{2}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{-\sqrt{3}}{2}$

(12) Which one of the following functions is the fastest growing as $x \to \infty$:

(a) $e^{\frac{x}{2}}$

(b) $\ln(\ln x)$

(c) 3^x

(d) $4 + 2^x$

(13) The series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n + 2^n}$:

(a) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(b) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(c) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{7^n}$

(d) Converges by summing its terms as a geometric series

(14) The series $\displaystyle\sum_{n=2}^{\infty} \frac{(n+1)\ln n}{\sqrt{n}}$:

(a) Converges by the integral test

(b) Converges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$

(c) Diverges by the ratio test

(d) Diverges by the nth-term test

(15) If $a_n = \left(1 - \frac{2}{n}\right)^{\frac{n}{2}}$, $n \in \mathbb{N}$, then $\lim_{n\to\infty} a_n =$

(a) e^{-2}

(b) e^{-1}

(c) e^{-4}

(d) $e^{\frac{-1}{2}}$

(16) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+2)(n+3)}}$:

(a) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^4}}$

(b) Converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(c) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

(d) Diverges by the ratio test

(17) $i^{215} =$

(a) i

(b) $-i$

(c) 1

(d) -1

(18) The integral $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}\,(\sqrt{x}-1)}$:

(a) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}}$

(b) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt{x}}$

(c) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x^2}}$

(d) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[6]{x^5}}$

(19) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{e^n(x-1)^n}{n^2 3^n}$ is:

(a) $\frac{3}{e}+1$

(b) $\frac{e}{3}+1$

(c) $\frac{3}{e}$

(d) $\frac{e}{3}$

(20) $\int_0^1 x^2 \ln x \, dx =$

(a) $\frac{-1}{4}$

(b) $\frac{-1}{9}$

(c) ∞

(d) $-\infty$

(21) The series $\sum_{n=1}^{\infty} \frac{(-1)^n (\sqrt{n}+n)}{\sqrt{n^5+1}}$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(d) Diverges by the nth-term test

(22) $\int_1^{\sqrt{3}} \frac{dx}{x\sqrt{x^2+1}} =$

(a) $\ln\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$

(b) $\ln\left(\frac{\sqrt{2}}{\sqrt{3}+1}\right)$

(c) $\ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right)$

(d) $\ln\left(\frac{\sqrt{3}}{\sqrt{2}+1}\right)$

(23) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2 x \tan^2 x \, dx =$

(a) $\dfrac{-26}{9\sqrt{3}}$

(b) $\dfrac{28}{9\sqrt{3}}$

(c) $\dfrac{-13}{3\sqrt{3}}$

(d) $\dfrac{20}{3\sqrt{3}}$

(24) A partial fraction for the function $f(x) = \dfrac{3x+1}{x^3-8}$ is:

(a) $\dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

(b) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+2}$

(c) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2-2x+4}$

(d) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2x+4}$

(25) The series $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{n^2-1}\right)^{n^2}$:

(a) Converges by summing its terms as a telescoping series

(b) Converges by the nth-term test

(c) Converges by the root test

(d) Diverges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n}}$

(26) $\dfrac{4-i}{1+i} =$

(a) $\dfrac{3}{2} - \dfrac{5}{2}i$

(b) $\dfrac{3}{2} + \dfrac{5}{2}i$

(c) $\dfrac{5}{2} - \dfrac{3}{2}i$

(d) $\dfrac{5}{2} + \dfrac{3}{2}i$

(27) The series $\sum_{n=3}^{\infty} \frac{\sqrt{\ln n}}{n^{1.1}}$:

(a) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$

(b) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.1}}$

(c) Converges by direct comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{0.95}}$

(d) Diverges by the ratio test

(28) If $x = \ln(\sec t + \tan t)$, $y = t \sec t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, then $\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} =$

(a) $\frac{\pi}{2} + 1$

(b) $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$

(c) $\frac{\pi}{4} + 1$

(d) None of the above

(29) The series $\sum_{n=1}^{\infty}(-1)^n \left(\sqrt{n+1} - \sqrt{n-1}\right)$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by the nth-term test

(d) Diverges by direct comparison with $\sum_{n=1}^{\infty}(-1)^n\sqrt{n}$

(30) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}} =$

(a) $\ln\left(\frac{2}{3}\right)$

(b) $\ln\left(\frac{1}{3}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln\left(\frac{3}{4}\right)$

Question 2. (15%) (a) Use the binomial series to find out the first four nonzero terms of the Maclaurin series of $(1+x)^{\frac{2}{3}}$, $-1 < x < 1$.

(b) (1) Find the Taylor series of $f(x) = \tan^{-1}(3x^2)$, about $a = 0$, and specify its interval of convergence.

(2) Use the above series to estimate the value of $\tan^{-1}\left(\dfrac{1}{3}\right)$ with an error of magnitude less than 0.001.

Question 3. (13%) (a) Find the length of the parametric curve:

$$x = t, \quad y = \frac{t^2}{2}, \quad 0 \leq t \leq 1.$$

(b) Sketch the parametric curve defined by the equations:

$$x = 3\cos t, \quad y = 2\sin t, \quad -\frac{\pi}{2} \leq t \leq \pi.$$

Question 4. (12%) (a) Find the four forth roots of -81.

(b) Solve the equation: $2|z-1-i| = |z+\overline{z}-2|$.

Birzeit University- Mathematics Department
Calculus II-Math 132

Final Exam — Spring 2013/2014

Name(Arabic):.. Number:..................

Instructor of Discussion(Arabic):................................ Section:..................

Question 1.(90.5%) Solve the following then circle the correct answer:

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$
   (a) Diverges by alternating series test.
   (b) Converges conditionally.
   (c) Converges absolutely.
   (d) Diverges by integral test.

2. $\int_1^e \frac{\ln x}{x^2} dx =$
   (a) $1-2e^{-1}$.
   (b) $\frac{1}{2}$.
   (c) $-e^{-1}\ln 2$.
   (d) $4e^{-1}+1$.

3. The first four terms of the binomial series of $(1+x)^{1/3}$ are
   (a) $1+x-\frac{1}{3}x^2+\frac{1}{81}x^3$.
   (b) $1+\frac{1}{6}x+\frac{1}{9}x^2+\frac{2}{81}x^3$.
   (c) $1+\frac{1}{3}x+\frac{1}{4}x^2+\frac{1}{5}x^3$.
   (d) $1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{81}x^3$.

4. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n x^n}{n}$ is
   (a) $[-1,1]$.
   (b) $[-2,2]$.
   (c) $(-\frac{1}{2},\frac{1}{2}]$.
   (d) $[-\frac{1}{2},\frac{1}{2})$.

5. The Maclaurin series generated by e^{x^2} is

(a) $\sum\limits_{n=0}^{\infty} \frac{x^{2n}}{n!}$.

(b) $\sum\limits_{n=0}^{\infty} \frac{x^{n}}{n!}$.

(c) $\sum\limits_{n=0}^{\infty} \frac{x^{n}}{(2n)!}$.

(d) $\sum\limits_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$.

6. $\int\limits_1^2 x^3(\ln x)dx =$

(a) $3\ln 2 + \frac{15}{16}$.

(b) $4\ln 2 - \frac{15}{16}$.

(c) $3\ln -\frac{5}{6}$.

(d) $3\ln 2 + \frac{5}{6}$.

7. The series $\sum\limits_{n=1}^{\infty} \frac{\sin^2 n}{n^{3/2}}$

(a) Converges by direct comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

(b) Diverges by nth tern test.

(c) Diverges by integral test.

(d) Converges conditionally.

8. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ converges to

(a) e^{-1}.

(b) e.

(c) 1.

(d) Diverges.

9. $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n} =$

(a) $\frac{10}{3}$.
(b) $\frac{5}{3}$.
(c) 2.
(d) $\frac{1}{3}$.

10. The sequence $a_n = \left(1 - \frac{2}{n}\right)^n$

(a) Converges to 1.
(b) Converges to e^{-2}.
(c) Converges to -2.
(d) Diverges.

11. The slope of the parametric curve $x = 2\cos t$, $y = 3\sin t$ at $t = \frac{\pi}{4}$ is

(a) $\frac{2}{3}$.
(b) $\frac{3}{2}$.
(c) $-\frac{2}{3}$.
(d) $-\frac{3}{2}$.

12. If $x = 1 + \cos\theta$, $y = 2 + \sin\theta$ then $\frac{d^2y}{dx^2}$ when $\theta = \frac{\pi}{4}$ is

(a) $-\sqrt{2}$.
(b) -1.
(c) $-2\sqrt{2}$.
(d) 2.

13. $\int_2^{\infty} \frac{dx}{x(\ln x)^{1/3}}$

(a) Converges to $2\ln 2$.
(b) Converges to $\frac{1}{\ln 2}$.
(c) Converges to $\frac{2}{\sqrt{\ln 2}}$.
(d) Diverges.

14. The sum of the telescoping series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ is

(a) 1.

(b) $\frac{1}{2}$.

(c) $\frac{3}{2}$.

(d) $\frac{2}{3}$.

15. $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}dx =$

(a) $\frac{2}{3}$.

(b) $\frac{20}{21}$.

(c) $\frac{8}{21}$.

(d) $\frac{2}{7}$.

16. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Diverges.

(b) Converges conditionally.

(c) Converges absolutely.

(d) None of the above.

17. $\int_0^1 \sin^{-1} x \, dx =$

(a) $\frac{\pi}{2} - 1$.

(b) $\frac{\pi}{2}$.

(c) 1.

(d) $\frac{\pi}{2} + 1$.

18. $\int_1^2 \frac{dx}{x(x+1)} =$

(a) $\ln 4 - \ln 3$.

(b) $-\ln 2$.

(c) $\ln 2 - \ln 3$.

(d) $\frac{1}{6}$.

19. $4^{2\log_4 2} =$

(a) 1.

(b) 4.

(c) $\ln 4$.

(d) 2.

20. The length of the curve $x = t$, $y = \cosh t$, $0 \leq t \leq 1$ is

(a) $\sinh 1$.

(b) $\cosh 1$.

(c) 1.

(d) e.

21. The series $\sum\limits_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$

(a) Converges by limit comparison test with $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$.

(b) Converges by limit comparison test with $\sum\limits_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

(c) Diverges by limit comparison test with $\sum\limits_{n=1}^{\infty} \frac{1}{n}$.

(d) Diverges by limit comparison test with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

22. The series $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right)$

(a) Diverges by nth term test.

(b) Converges by integral test.

(c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)$.

(d) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

23. The sequence $a_n = \ln(n+2) - \ln(n+1)$

(a) Converges to 1.

(b) Converges to 0.

(c) Converges to e^{-1}.

(d) Diverges.

24. The sequence $a_n = n \sin\left(\frac{\pi}{n}\right)$

(a) Converges to π.

(b) Converges to 0.

(c) Converges to 1.

(d) Diverges.

25. Using alternating series estimation, we can approximate $\cos x$ by $1 - \frac{x^2}{2}$ with error less than 0.01 if

(a) $|x| < 0.01$.

(b) $|x| < \sqrt{(4!)(0.01)}$.

(c) $|x| < \sqrt[4]{(4!)(0.01)}$.

(d) $|x| < \sqrt[4]{(0.01)}$.

26. The series $\sum\limits_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

(a) Converges to 0.

(b) Converges to $\ln(1/2)$.

(c) Diverges by nth term test.

(d) Diverges by alternating series test.

27. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(a) Converges by ratio test.

(b) Diverges by ratio test.

(c) Diverges by root test.

(d) Converges by direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{n!}$.

28. $\left(\frac{1+i}{1-i}\right)^2 =$

(a) 1.

(b) i.

(c) $-i$.

(d) -1.

29. The functions $x^2, e^x, 10^x, \ln x$ from slowest to fastest are

(a) $\ln x, x^2, 10^x, e^x$.

(b) $x^2, \ln x, e^x, 10^x$.

(c) $x^2, \ln x, 10^x, e^x$.

(d) $\ln x, x^2, e^x, 10^x$.

30. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{n}$ is

(a) 1.

(b) $\frac{1}{2}$.

(c) 4.

(d) 2.

31. The error in the estimation $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ in the interval $[0, 0.1]$ using Taylor's theorem is less than

(a) 8×10^{-2}.

(b) 4×10^{-2}.

(c) $\frac{1}{8} \times 10^{-2}$.

(d) None of the above.

32. One of the following is <u>false</u>

(a) The series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges by nth term test.

(b) The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by integral test.

(c) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{5/4}}$ converges conditionally.

(d) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$ converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

33. $\int_0^1 \frac{dx}{\sqrt{1+x^2}} =$

(a) $\ln\sqrt{2}$.

(b) $\ln(1+\sqrt{2})$.

(c) $\ln 2$

(d) None of the above.

34. $\int_1^e 4\cosh(\ln x)dx =$

(a) e.

(b) e^2.

(c) $1+e$.

(d) $1+e^2$.

35. $\int_1^e \frac{\log_4 t}{t} dt =$

(a) $\ln 4$.

(b) $\frac{1}{2\ln 4}$.

(c) $\frac{1}{4}$.

(d) None of the above.

36. The integral $\int_2^\infty \frac{dx}{x+\ln x}$

(a) Converges to 1.

(b) Diverges by direct comparison with $\int_2^\infty \frac{dx}{x}$.

(c) Diverges by limit comparison with $\int_2^\infty \frac{dx}{x}$.

(d) Diverges by direct comparison with $\int_2^\infty \frac{dx}{\ln x}$.

37. The parametric curve $x = \cos^2 t, y = \sin^2 t,\ 0 \le t \le \frac{\pi}{2}$ presents

(a) The line segment from $(0,1)$ to $(1,0)$.

(b) The line segment from $(0,1)$ to $(-1,0)$.

(c) The line segment from $(1,0)$ to $(0,1)$.

(d) The unit circle.

Question 2(8%) Find the six sixth roots of -1. Write all roots in the form of $a + ib$.

12/20

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 - THIRD EXAM-
SUMMER 2013/2014

- Name...[redacted]......
- Number.[redacted]........

- (For Question 1) Fill your answers in the tables below:

Page 1	
1	b
2	d
3	d
4	a
5	c

Page 2	
6	a
7	d
8	d
9	a
10	b

Page 3	
11	b
12	a

- Instructions:
  1. No Calculators.
  2. Mobiles Off.
  3. BZU ID On Your Desk.
  4. No Cheating At All.
  5. Time Limit: 60 Minutes.

Question 1. *(12 points)* Circle the best answer.

Root test $= \sum$

1. The series $\sum_{n=1}^{\infty}\left(\frac{4n+3}{3n-2}\right)^n$

$\lim_{n\to\infty}\left(\frac{4n+5}{3n-2}\right)^{n/n}$

   a) Converges by root test
   b) Diverges by root test
   c) Converges by integral test
   d) Diverges by alternating series test

$\boxed{\frac{4}{3}>1}$

2. If we approximate e^x by $1+x+\frac{x^2}{2!}$, then the error in estimating e^{-1} is

$\boxed{x=-1}$ $\boxed{a=0}$ $n=2$

$e^x = 1+x+\frac{x^2}{2!}$ $e^x = \frac{x^n}{n!}$

   a) less than $\frac{1}{2}$
   b) less than $\frac{1}{2e}$
   c) less than $\frac{1}{6}$
   d) less than $\frac{1}{e}$

$\left|\frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}\right| < \frac{M(x-a)^{n+1}}{(n+1)!}$

$a<c<x$

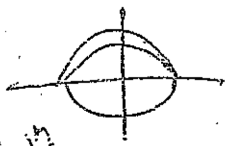
$0<c<-1$

$e^c < e^{-1}$

$\left|\frac{e^c (x)^3}{3!}\right| < \frac{e^{-1}(x)^3}{3!}$

3. The radius of convergence of the series $\sum_{n=0}^{\infty}(n+1)!\,(x-4)^n$ is

   a) $R=0$
   b) $R=1$
   c) $R=4$
   d) $R=\infty$



Ratio Test

$\lim_{n\to\infty}\left|\frac{(n+1+1)!\,(x-4)^{n+1}}{(n+1)!\,(x-4)^n}\right|$

$\lim_{n\to\infty}\left|\frac{(n+2)\cancel{(n+1)!}\,\cancel{(x-4)^n}(x-4)}{\cancel{(n+1)!}\,\cancel{(x-4)^n}}\right|$

$|x-4|$

4. The series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{\sqrt{n}}$

$= \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

$-1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}$

   a) Converges absolutely
   b) Converges conditionally
   c) Diverges by alternating series test
   d) Diverges by nth term test

$\frac{1}{\sqrt{n}}$ $a_n = \frac{1}{\sqrt{n}}$ $u_n > 0$ decreasing

$\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$

5. $1+\pi+\frac{\pi^2}{2!}+\frac{\pi^3}{3!}+\cdots=$

   a) 0
   b) -1
   c) e^{π}
   d) None of the above

$\cos 0 = 1$
$-\sin = 0$
$-\cos 0 = -1$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ $-1 + x - \frac{x^3}{3!}$

6. The series $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n^5}}$

a) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

c) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

d) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n}}$

$\lim_{n\to\infty} 1 = 1$, $1 \neq 0$, diverges

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally if

a) $0 < p < 1$

b) $0 \le p < 1$

c) $0 < p \le 1$

d) $0 \le p \le 1$

$\lim_{n\to\infty} \dfrac{\frac{\ln n}{\sqrt{n^5}}}{\frac{1}{\sqrt{n^5}}} = \infty$

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

a) Converges by integral test

b) Diverges by integral test

c) Converges by nth term test

d) None of the above

$\frac{5}{2} > 1$ Converges

9. The binomial series of $\frac{1}{\sqrt{1+x}}$ is

a) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

b) $1 + \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

c) $1 - \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} - \cdots$

d) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{x^3}{16} + \cdots$

$(1+x)^{-\frac{1}{2}}$ $m = -\frac{1}{2}$ $1 + \sum_{n=1}^{\infty} \binom{m}{n} x^n$

10. The Maclaurin series generated by $x \sin x^2$ is

a) $x^3 + \frac{x^7}{3!} - \frac{x^{11}}{5!} + \cdots$

b) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \cdots$

c) $x + \frac{x^5}{2!} - \frac{x^9}{4!} + \cdots$

d) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \cdots$

$\lim_{n\to\infty} \frac{1}{\ln n} \Rightarrow 0$

$\frac{3}{4}x^2$

11. The Taylor polynomial of order 3 generated by $f(x) = e^{2x}$ about $a = 0$ is

a) $P_3(x) = 1 + 2x + x^2$

b) $P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$

c) $P_3(x) = 1 + x + 2x^2 + \frac{4}{3}x^3$

d) $P_3(x) = 1 + x + x^2$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \frac{(2x)^n}{n!}$$

0 1 2

$$1 + 2x + \frac{\cancel{4}x^2}{\cancel{2}}$$

$$1 + 2x + 2x^2 + \frac{\cancel{48}x^3}{\cancel{36}}$$

12. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

a) Converges by ratio test

b) Diverges by ratio test

c) Converges to 4

d) Converges by root test

$$\frac{(2n+1)!}{(n+1)!(n+1)!} \cdot \frac{n!n!}{2n!}$$

$$\frac{1}{n^2} = \boxed{0} < 1 \quad \lim_{n\to\infty} \frac{(2n+1)}{(n+1)(n+1)}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Question 2. *(4 points)* Given that $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

$$\Rightarrow 1 - \cancel{\frac{x^2}{2!}} + \cancel{\frac{x^4}{4!}}$$

(a) Find the Maclaurin series of $\cos x^3$.

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

2

(b) Use part (a) to estimate $\int_0^1 \cos x^3 \, dx$ with error less than 0.01

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} = \int_0^1 1 + -\frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!}$$

$$= \left. x - \frac{x^7}{7(2!)} + \frac{x^{13}}{13(4!)} - \frac{x^{19}}{19(6!)} \right]_0^1$$

1

$$\frac{\cdots}{(1+x^2)}$$

Question 3. *(2 points)* Express $\dfrac{1}{(1+x)^2}$ as a power series ~~and~~ find its radius of convergence.

(Hint: $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$)

$$\cancel{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n} \quad \cancel{\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n x^n}$$

$$\cancel{(-1)^n (nx)^{n-1}} \quad \cancel{\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n}$$

$$\cancel{\frac{1}{(1+x)^2}}$$

$$\frac{d}{dx}\left(\frac{1}{1+x}\right) = 1 - x + x^2 - x^3 + \cdots$$

$$\frac{-1}{(1+x)^2} = -1 + 2x - 3x^2$$

$$\frac{-1}{1+x^2} = \sum_{n=1}^{\infty} (-1)^n (nx)^{n-1} \cdots$$

$$\frac{1}{1+x^2} \neq \sum_{n=1}^{\infty} (nx)^{n-1}$$

by Ratio Test

$$\lim_{n\to\infty} \left| \frac{((n+1)x)^n}{(nx)^{n-1}} \right| \Rightarrow \left| \frac{(n+1)^n x^n}{n^{n-1} x^{n-1}} \right|$$

$$|x| \lim_{n\to\infty} \frac{(n+1)^n}{n^{n-1}}$$

Question 4. *(2 points)* Use series to find $\displaystyle\lim_{x\to 0} \frac{\sin x}{e^{-x} - 1}$.

$$\sin x = \sum_{n=0}^{\infty} x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{-x} = \frac{(-1)^n (x^n)}{n!}$$

$$= 1 - x + \frac{x^2}{2!}$$

$$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{\left(1 - x + \frac{x^2}{2!} - 1\right)} \quad \cancel{x\left(1 - \frac{x^2}{3!} + \frac{x^5}{}\right)}$$

BONUS. *(2 points)* Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{x^n n!}{(1)(3)(5)\ldots(2n-1)}$.

$$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)}$$

1 - 5

$$\lim_{x\to 0} \frac{x\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right)}{x\left(-1 + \frac{x}{2!} - \frac{x^2}{3!} - \cdots\right)} = -1$$

Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Marwan Awartani

Fall [illegible]

Final Exam

Student Name:______________________ Number:________ Section: ____

Q1: (60 points) Circle the MOST correct answer:

1. $\int_e^{e^2} \frac{1}{x \ln x} dx =$

   $u = \ln x$
   $du = \frac{dx}{x}$ $\quad dx = x\,du$
   $= \int_e^{e^2} \frac{du}{u} = \ln|u| \Big]_e^{e^2} = \ln|\ln x| \Big]_e^{e^2}$
   $= \ln \ln e^2 - \ln \ln e$
   $= \ln 2$

   (a) 1
   (b) $\ln 2$
   (c) $\ln\left(\frac{1}{2}\right)$
   (d) 0

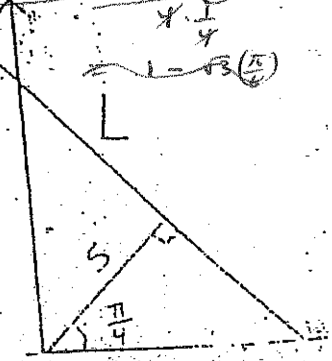
2. The curve with parametric equations $x = \sin t$, [illegible] $-\infty < t < \infty$.
   (a) A segment of a parabola
   (b) A circle
   (c) An ellipse
   (d) A hyperbola

3. The slope of the curve $x = \sin 2t$, $y = \cos t$ at $t = +\frac{\pi}{6}$ is:

   $x = 2 \sin t \cos t$
   $x = 2 \sin\frac{\pi}{6} \cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2}\right) = \frac{\sqrt{3}}{2}$
   $x = 2 \sin t\, y$
   $y = \frac{x}{2 \sin t} \Rightarrow y' = \frac{2 \sin t - 2y \cos t (x)}{4 \sin^2 t} = \frac{2 \cdot \frac{1}{2} - 2 \cdot \frac{\sqrt{3}}{2} \left(\frac{\pi}{6}\right)}{4 \cdot \frac{1}{4}}$
   $= 1 - \sqrt{3}\left(\frac{\pi}{6}\right)$

   (a) 0
   (b) 1
   (c) $\frac{1}{2}$
   (d) $-\frac{1}{2}$

4. The polar equation of straight line L is:
   (a) $r \cos\left(\theta + \frac{\pi}{4}\right) = -5$
   (b) $r \cos\left(\theta - \frac{\pi}{4}\right) = 5$
   (c) $r \sin\left(\theta + \frac{\pi}{4}\right) = -5$
   (d) $r \cos\theta = 5$



5. The graphs of the curves with polar coordinates $r = \sin\theta$, $r = -\cos\theta$ intersects at:

(a) Only at the origin.
(b) Only when $\tan\theta = -1$
(c) At exactly two points.
(d) At exactly three points.

? 6. The equation of $x^2 + 5xy + y^2 = 3$ is

(a) Circle
(b) Ellipse
(c) A hyperbola
(d) A parabola

7. One of the following is not an improper integral

(a) $\int_0^{10} \frac{\sin x}{x}dx$ $= (x - \frac{x^3}{3!}$ - - -

(b) $\int_{-\infty}^{a} x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}}dx$

(d) $\int_0^\infty \frac{dx}{x^2 - 1}$

$du = \cos x . dx$
$u = \sin x$

8. $\int \sin^2 x \cos^3 x dx =$ $= \int \sin^2 x \cos^2 x \cos x \, dx$
$= \int u^2(1-u^2) . du = \int (u^2 - u^4) . du$
$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{15} + c$

(d) $\frac{\sin^3 x}{15}(5 + 3\sin^2 x) + c$

9. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1 - t^4}$, $-1 \le t \le 1$, then

(a) The initial point is (1,0) and the end point is (0,1).
(b) the motion is clockwise.
(c) the motion is counter clockwise.
(d) None of the above.

? 10. $\lim_{x \to 0^+} (1 + x)^{\frac{1}{x}}$

$x \to 0 \Rightarrow u \to \infty$

(a) e
(b) 1
(c) $\frac{1}{e}$
(d) 0

10. The graph of the curves with polar coordinates $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$ intersects in

(a) One point only
(b) Two points only
(c) Three points only
(d) Four points only

11. The slope of the polar curve $r = 1 + 2\cos\theta$ at the origin is

(a) 1
(b) -1
(c) $\sqrt{3}$
(d) $\pm\sqrt{3}$

$x = 0$
$u = 0$

$u = \tan^{-1} x$
$x = \tan u$
$dx = \sec^2 u \, du$

? 12. $\lim_{x \to 0^+} (1 + x)^{\frac{1}{x}} =$

(a) e
(b) 1
(c) $\frac{1}{e}$
(d) 0

$\lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{u \to 0} \frac{u}{\tan u} = \lim_{u \to 0} \frac{u \cos u}{\sin u} =$

$x \to 0$
$u \to 0$

$x = \tan u$

$u = \tan^{-1} x$
$du = \frac{dx}{1+x^2}$

$dx = (1+x^2) \cdot du$

? 13. $\lim_{x \to 0} \frac{\tan^{-1} x}{x} =$

(a) 1
(b) -1
(c) 0
(d) ∞

$= \lim_{x \to 0} \frac{(1+x^2)}{x} \cdot du = \frac{1 + \tan^2 u}{\tan u} \cdot du$

$= \int \frac{\sec^2 u}{\tan u} \cdot du$

$= \int \frac{1}{\cos^2 u} \cdot \frac{\cos u}{\sin u} \cdot du = \int$

14. [illegible] =

(a) [illegible]
(b) $\frac{7}{5}$
(c) 5
(d) 7

15. The eccentricity of the conic section $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

(a) $\frac{5}{3}$
(b) $\frac{5}{4}$
(c) $\frac{3}{4}$
(d) $\frac{4}{3}$

$a = 3$ $b = 4$
$c^2 = a^2 + b^2$
$= 9 + 16$
$c = 5$

$e = \frac{c}{a}$
$= \frac{5}{3}$

16. The length of the polar curve $r = 4\cos\theta$, $0 \le \theta \le \frac{\pi}{2}$ is

(a) π
(b) 2π
(c) 3π

17. The surface area generated by revolving $r = 4\cos\theta \;\; 0 \le \theta \le \frac{\pi}{2}$ about x-axis is

(a) 2π
(b) 4π
(c) 8π
(d) 16π

18. The Cartesian coordinates of the point $P(-4, \frac{\pi}{4})$ is

(a) $(-2\sqrt{2}, 2\sqrt{2})$
(b) $(-2\sqrt{2}, -2\sqrt{2})$
(c) $(2\sqrt{2}, 2\sqrt{2})$
(d) $(2\sqrt{2}, -2\sqrt{2})$

19. The angle θ that eliminate xy term in $2x^2 + \sqrt{3}xy + y^2 - 2y = 6$ is

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

20. $\sec^{-1}(-\sqrt{2}) =$

$-\sqrt{2} = \sec y$

$-\sqrt{2} = \frac{1}{\cos y}$

$\cos y = \frac{-1}{\sqrt{2}}$

$y = \frac{\pi}{4} + \frac{\pi 4}{4} = \frac{5\pi}{4}$

$\frac{\pi}{4} - \frac{\pi 4}{4} = -\frac{3\pi}{4}$

21. $\frac{\pi}{4}$
22. $\frac{-\pi}{1}$
23. $\frac{3\pi}{4}$
24. $\frac{5\pi}{4}$

II (15 points): Evaluate the following integrals

$\int \sin^{\frac{5}{2}} x$

Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Hasan Yousef

Summer

Final Exam

Student Name:________________________ **Number:**________ **Section**____

I (40points) : Circle the MOST correct answer:

1. If $\sinh x = \frac{-3}{4}$ then $\cosh x =$

   y -3 5

   (a) $\frac{3}{5}$
   (b) $\frac{5}{4}$
   (c) $\frac{-3}{5}$
   (d) $\frac{-5}{4}$

2. The conic section with Foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$ is an

   $c = 1$ $a = 2$ $a > c$ $e < 1$

   (a) Ellipse
   (b) Parabola
   (c) Hyperbola
   (d) A circle

3. The conic section with eccentricity $\frac{1}{2}$ and directrix $x = 2$ has equation

   $e = \frac{c}{a} = \frac{1}{2}$ $\frac{a}{e} = 2$ $\frac{a}{2} = \frac{c}{a} = \frac{1}{2}$ $\boxed{e = \frac{a}{2}}$ $a^2 = 2c$ $\boxed{a = 1}$ $2c = a$ $a = 1$

   $D = \frac{a}{e} = 2$ $\boxed{a = 2e}$ $e = \frac{c}{a} = \frac{1}{2}$ $\boxed{a = 1}$ $a = 2c \to \boxed{c = \frac{1}{2}}$ $b^2 = a^2 - c^2 = 1 - \frac{1}{4}$ $\boxed{b = \frac{\sqrt{3}}{2}}$ $= \frac{3}{4}$

   $a = 2e = 2 \cdot \frac{1}{2} = 1$ $c = ea = \frac{1}{2} \cdot 1$ $\boxed{c = \frac{1}{2}}$

   (a) $2x^2 + y^2 = 1$
   (b) $x^2 - y^2 = 1$
   (c) $y^2 - x^2 = 2$
   (d) $x^2 + \frac{4}{3}y^2 = 1$

   $\frac{x^2}{1} + \frac{4y^2}{3} = 1$

4. The directric of the parabola $x = \frac{y^2}{2}$ is given by

   $y^2 = 2x$ $4p x = 2x$ $\boxed{p = \frac{1}{2}}$

   (a) $x = 1$
   (b) $y = \frac{-1}{2}$
   (c) $y = \frac{1}{2}$
   (d) $x = \frac{1}{2}$

? 5. The conic section $x^2 + 4xy + \sqrt{2}y^2 + 5 = 0$ is

(a) Ellips
(b) Parabola
(c) Hyperbola
(d) A Circle

$x^2 + \sqrt{2}\, y^2 = -4xy - 5$

$\frac{x^2 + \sqrt{2}\, y^2}{-4xy-5} = \frac{-4xy - 5}{-4xy-5}$

$= \frac{x^2}{-4y} + \frac{\sqrt{2}\, y}{-4x} = \frac{5}{-4xy}$

$\cancel{-4x^2 - 4\sqrt{2}y^2} = \frac{5}{-4xy}$

? 6. $x = \cos 2t,\ \ y = \sin^2 t\ ,\ 0 \le t \le \frac{\pi}{4}$ represents

(a) half of a circle
(b) half of an Ellipse
(c) a line seqment
(d) a parabola

? 7. The slope of the Ellipse $x = 2\sin t,\ y = 3\cos t \quad 0 \le t \le 2\pi$ at the point $\left(\sqrt{2}, \frac{3}{\sqrt{2}}\right)$ is

(a) $-\frac{2}{3}$
(b) 3
(c) $\frac{3}{2}$
(d) $\frac{-3}{2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{4\sin^2 t}{a^2} + \frac{9\cos^2 t}{b^2} = 1$

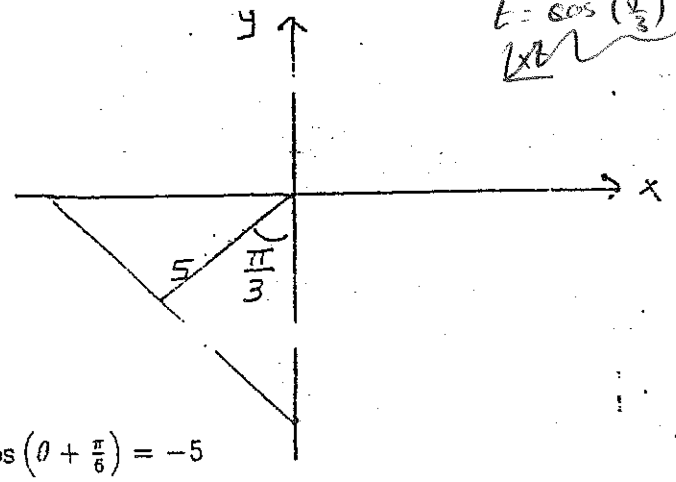
$t = \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{4}$

$t = \cos^{-1}\left(\frac{y}{3}\right) = \frac{\pi}{4}$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

8. An equation of the line in the figure is



(a) $r\cos\left(\theta + \frac{\pi}{6}\right) = -5$
(b) $r\sin\left(\theta + \frac{2\pi}{3}\right) = -5$
(c) $r\cos\left(\theta + \frac{5\pi}{6}\right) = 5$
(d) $r\sin\left(\theta + \frac{\pi}{6}\right) = -5$

9. The polar Equation of the circle with center $P(-2, \frac{\pi}{4})$ and radius 4 is

(a) $r = -4\sin\left(\theta + \frac{\pi}{4}\right)$
(b) $r = -4\sin\left(\theta + \frac{3\pi}{4}\right)$
(c) $r = 4\sin\left(\theta - \frac{3\pi}{4}\right)$
(d) $r = 4\sin\left(\theta - \frac{\pi}{2}\right)$

$$= \int \frac{x^{\cancel{2}} \cdot du}{2\cancel{x}\sqrt{1+x^2}}$$

$u = x^2$
$du = 2x \cdot dx$
$dx = \frac{du}{2x}$

$$= \int \frac{x \cdot du}{2\sqrt{1+x^2}} = \int \frac{\sqrt{u} \cdot du}{2\sqrt{1+u}}$$

$u = 1+x^2$
$du = 2x \cdot dx$
$dx = \frac{du}{2x}$

Q2:(15 points) (a) $\int \frac{x^2 dx}{\sqrt{1+x^2}} =$

$$\cancel{= \int \frac{(u-1) \cdot du}{2x}} = \int \frac{x^2 \cdot du}{2x\sqrt{u}} = \int \frac{\sqrt{u-1} \cdot du}{2\sqrt{u}}$$

$$\cancel{= \frac{1}{2}\int \sqrt{\frac{u-1}{u}} \cdot du = \frac{1}{2}\int \sqrt{1-\frac{1}{u}} \cdot du}$$

$u = \sqrt{1+x^2}$
$du = \frac{\cancel{2}x \cdot dx}{\cancel{2}\sqrt{1+x^2}}$

$$= \int \frac{x^{\cancel{2}} \cdot \cancel{\sqrt{1+x^2}} \cdot du}{\cancel{x}\ (\cancel{u})} = \int \sqrt{u^2-1} \cdot du$$

$du = \frac{x \cdot dx}{\sqrt{1+x^2}}$

$dx = \frac{\sqrt{1+x^2} \cdot du}{x}$

$$= \int (u^2-1)^{\frac{1}{2}} \cdot du$$

$u = \sqrt{1+x^2}$
$u^2 = 1+x^2$
$x = \sqrt{u^2-1}$

(b) $\int \frac{x^2+x+1}{x^3+x} dx =$

$$\frac{x^2+x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$$

? (c) $\lim_{x\to\infty} \left(\frac{x-1}{x-2}\right)^x = \lim_{x\to\infty} \frac{(x-1)^x}{(x-2)^x}$ $= \lim_{x\to\infty}$

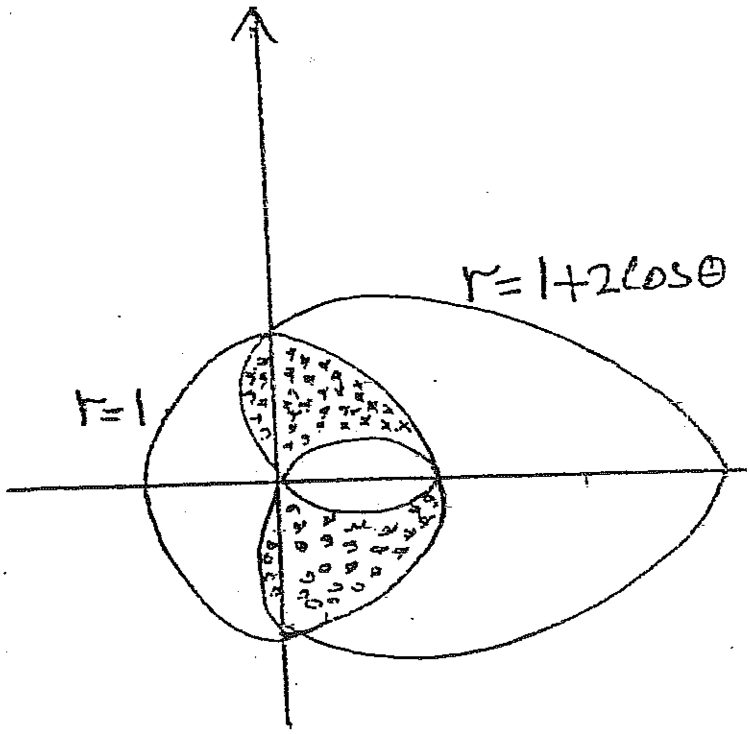
$$= \lim_{u\to\infty} \frac{(u)^{u+1}}{(u)^{u+2}} = \lim_{u\to\infty} u^{(u+1-u-2)}$$

$u = x-1$
$du = dx$
$x = u+1$
$x = \infty \quad u = \infty$

$$= \lim_{u\to\infty} u^{-1}$$

$$= \lim_{u\to\infty} \frac{1}{u} = 0$$

IV (15 points): Find the area of the shaded region.



Birzeit University
Math. & Comp. Science Dept.
Math. 132

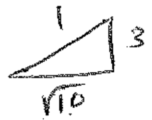
Final Exam **Second Semester** [redacted]

The Name of Discussion Teacher:

Student Name:______________________________ **Number:**_________ **Section**___

I (50 points) : Circle the MOST correct answer:

1. If $\sinh x = 3$ then $\cosh(-x) =$
   (a) $\sqrt{10}$
   (b) $-\sqrt{10}$
   (c) $\sqrt{8}$
   (d) $\sqrt{10}, -\sqrt{10}$



? 2. $\int \frac{ax+b}{x^3+x^2}dx$ is a rational function if

   (b) $b = 0$
   (c) $a = b$
   (d) $a = -b$

3. If $2^{x^2} = 4.2^x$ and $x > 0$ then $x =$
   (a) 2
   (b) -1
   (c) $2, -1$
   (d) 4

$x^2 \ln 2 = x \ln 4.2$

$x = \frac{\ln 4.2}{\ln 2} = \ln 2.2$

4. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}dx =$
   (a) $\tanh^{-1}(1)$
   (b) $\ln(e^2+1) - \ln 2$
   (c) $\ln(e^2+1) - \ln 2 - 1$
   (d) None of the above

$u = e^x$
$du = e^x \, dx$

$x = 1 \to u = e$
$x = 0 \to u = 1$

$= \int_1^e \frac{\left(u - \frac{1}{u}\right) du}{u\left(u + \frac{1}{u}\right)} = \int_1^e \frac{\frac{u^2-1}{u}}{u\left(\frac{u^2+1}{u}\right)} du = \int_1^e \frac{u^2-1}{u(u^2+1)} du = \int_1^e$

$\frac{u^2-1}{u(u^2+1)} = \frac{A}{u} + \frac{B}{u^2+1}$

$u^2 - 1 = A(u^2+1) + B(u)$
$u = 0$
$A = 1$
$u = 1$
$2A + B = 0$
$B = -2A$
$B = 2$

$= \int_1^e \left(\frac{1}{u} - \frac{2}{u^2+1}\right) du$

$= \ln u - 2\ln(u^2+1)\Big]_1^e$

$= \ln e - 2\ln(e^2+1) - \ln 1 + 2\ln(2)$

$= 1 - 2\ln(e^2+1) + 2\ln 2$

$= 2x\ln x - \frac{2(ue^u - e^u)}{\ln 3}$

$f(x) = u \quad g(x) = e^u$
$f'(x) = 1 \quad \int g(x) = e^u$
$\iint g(x) = e^u$

$= \int \frac{\ln x^2}{\ln 3} \cdot dx = \frac{1}{\ln 3} \int \ln x^2 \cdot dx$

$u = \ln x^2$

$= \frac{1}{\ln 3} \int u \cdot \frac{x}{2} \cdot du$

$du = \frac{2x}{x^2} \cdot dx$

5. $\int \log_3 x^2 dx =$

(a) $2x \log_3 x^2 - x + c$

(b) $2x \log_3 x - x + c$

(c) $\frac{2}{\ln 3}(x \ln x + x) + c$

(d) $\frac{2}{\ln 3}(x \ln x - x) + c$

$du = \frac{2}{x} \cdot dx$

$= \frac{1}{2\ln 3} \int u \cdot e^u \cdot du$

$dx = \frac{x}{2} \cdot du$

$z = e^u \quad dv = u\,du \quad x = e^{\frac{u}{2}}$
$dz = e^u\,du \quad v = \frac{u^2}{2} \quad \frac{u}{2} = \ln x$

? 6. If $\int_0^b f(x)dx$ diverges and $\int_a^b g(x)dx$ diverges then $\int_a^b f(x).g(x)dx$

(a) converges always

(b) diverges always

(c) Can't decide

(d) Converges if $f(x) = g(x)$

$= \frac{e^u u^2}{2} - \int \frac{u^2}{2} e^u \cdot du$

7. A line $y = ax + b$ and a curve of a function $y = e^x$ can intersect in at most

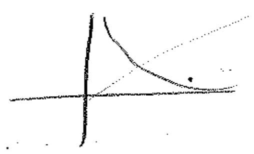
(a) 1 point

(b) 2 points

(c) 3 points

(d) 4 points

~~$ax + b = e^x$~~



8. If $\sin^{-1} x = \frac{\pi}{3}$ then $\cos^{-1} x =$

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{-\pi}{3}$

(d) $\frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

$x = \sin u$

$u = \sin^{-1} x$
$du = \frac{dx}{\sqrt{1-x^2}}$
$dx = \sqrt{1-x^2} \cdot du$
$dx = \sqrt{1-\sin^2 u} \cdot du$
$= \cos u \cdot du$

9. $\int_0^{\frac{\pi}{2}} \sin x\,dx + \int_0^1 \sin^{-1} x\,dx = -\cos x \Big]_0^{\pi/2} + \sin u \Big]_0^{\pi/4}$

$= 0 + 1 + \frac{1}{\sqrt{2}} - 0$

$\left(\frac{2+1}{\sqrt{2}}\right) \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}$

$x = 1$	$x = 0$
$u = \pi/4$	$u =$

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) $\frac{1}{2}$

10. One of the following is not an improper integral.

(a) $\int_0^{10} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^{\pi} x\,dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int^{\infty} \frac{dx}{x^2 - 1}$

11. $\int \sin^2 x \cos^3 x \, dx =$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{12} + c$

(d) $\frac{\sin^3 x}{15}(5 + 3\sin^2 x) + c$

12. The function $f(x) = \ln[x + 2]$ has domain

(a) $(-2, \infty)$

(b) $(0, \infty)$

(c) $[-1, \infty)$

(d) $[0, \infty)$

13. $\int x^3 e^x dx =$

(a) $\frac{x^4}{4} e^x + c$

(b) $\frac{x^4}{4} e^x - e^x + c$

(c) $e^x(x^3 - 3x^2 + 6x - 6) + c$

(d) $e^x(-x^3 + 3x^2 - 6x + 6) + c$

14. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1 - t^4}$, $0 \le t \le 1$, then

(a) The initial point is $(1, 0)$ and the end point is $(0, 1)$.

(b) The motion is clockwise.

(c) The motion is counter clockwise.

(d) [illegible]

15. The total distance travelled by a particle moving on the curve $r = 2\sin\theta$ from $\theta = 0$ to $\theta = 2\pi$ is

(a) 4π

(b) 2π

(c) π

(d) 8π

16. The slope of the tangent lines to the curve $r = 1 - 2\cos\theta$ at the origin are

(a) 0

(b) $\pm\sqrt{3}$

(c) ± 1

(d) not defined

$y = \pm \frac{b}{a} x$

$a = 4 \quad c = 5$

$b = 3$

$e = \frac{c}{a} = \frac{5}{4}$

$x = \frac{a}{e}$

17. The directrices of the hyperbola $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$ are

(a) $y = \pm\frac{16}{5}$

(b) $x = \pm\frac{16}{5}$

(c) $x = \frac{-26}{5}, x = \frac{6}{5}$

(d) $y = \frac{-26}{5}, y = \frac{6}{5}$

$D = \frac{a}{e} = \frac{4}{5} \cdot 4 = \frac{16}{5}$

$\frac{4}{\frac{5}{4}}$

18. The polar curves $r = \cos 2\theta, r = \frac{1}{2}$ intersects in

(a) 1 point

(b) 2 points

(c) 4 points

(d) 8 points

$x \pm \frac{16}{5}$

$x = +\frac{16}{5} - 2 \qquad x = -\frac{16}{5} \qquad 2$

19. The circles $x^2 + y^2 = 1$ and $(x-1)^2 + y = a^2$ intersect in two point if:

(a) $a = 2$

(b) $2 < a$

(c) $a < 2$

(d) $0 < a < 2$

$x \to 0$

$u \to \infty$

20. $\lim\limits_{x \to 0^+} \frac{\ln x}{\sqrt{x}} =$

$u = \ln x$

$du = \frac{dx}{x}$

$dx = x \ln x$

$x = e^4 \qquad u = \sqrt{x}$

(a) 0

(b) $-\infty$

(c) ∞

(d) 1

$= \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \frac{2\sqrt{x}}{x} = \infty$

?? II (15 points): 1. $\lim\limits_{x \to \infty} \left(\frac{x-1}{x-2}\right)^x$

$= \lim\limits_{x \to \infty} \frac{(x-1)^x}{(x-2)^x} = \frac{x \ln(x-1)}{x \ln(x-2)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots}{\ln 2 - \frac{x}{2} - \frac{x^2}{4}}$

$f(x) = \ln(x-2) \quad f(0) = \ln 2$

$f'(x) = \frac{1}{x-2} \quad f'(0) = -\frac{1}{2} \qquad = \ln 2$

$f''(x) = \frac{-1}{(x-2)^2} \quad f''(0) = -\frac{1}{4}$

2. Test for convergence $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

$u = e^x$
$du = e^x \, dx$
$dx = \frac{du}{e^x} = \frac{du}{u}$

$= \int_{-\infty}^{\infty} \frac{du}{u(u+\frac{1}{u})} \quad = \int_{-\infty}^{\infty} \frac{du}{\cancel{u}(\frac{u^2+1}{\cancel{u}})}$

$= \tan^{-1} u \Big]_{-\infty}^{0} + \tan^{-1} \Big]_{0}^{\infty}$

$= \tan^{-1}(e^x) \Big]_{-\infty}^{0} + \tan^{-1}(e^x) \Big]_{0}^{\infty}$

$= \cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

? 3. Evaluate $\int \frac{\sqrt{9 - x^2}}{x} dx$.

~~$u = 9 - x^2$~~
~~$du = -2x \, dx$~~
~~$dx = -\frac{du}{2x}$~~

~~$\int \frac{\sqrt{u}}{2x^2} du$~~

~~$= \int \frac{\sqrt{u}}{2(9-u)} du$~~

~~$= \int \frac{\sqrt{9-u}}{2u} du$~~

$u = x^2$
$du = 2x \, dx$
$dx = \frac{du}{2x}$

~~$\sqrt{a^2 - x^2}$~~ $\quad x = a \sin\theta$

$a^2 - x^2 \quad x = a \sin\theta$
$dx = a \cos\theta \, d\theta$

$= \int \frac{\sqrt{a^2 - a^2 \sin^2\theta}}{\cancel{a} \sin\theta} a \cos\theta \, d\theta$

$= \int \frac{\cos^2\theta \, d\theta}{\sin\theta} = \int \cot\theta \csc\theta \, d\theta = -\csc\theta + C$

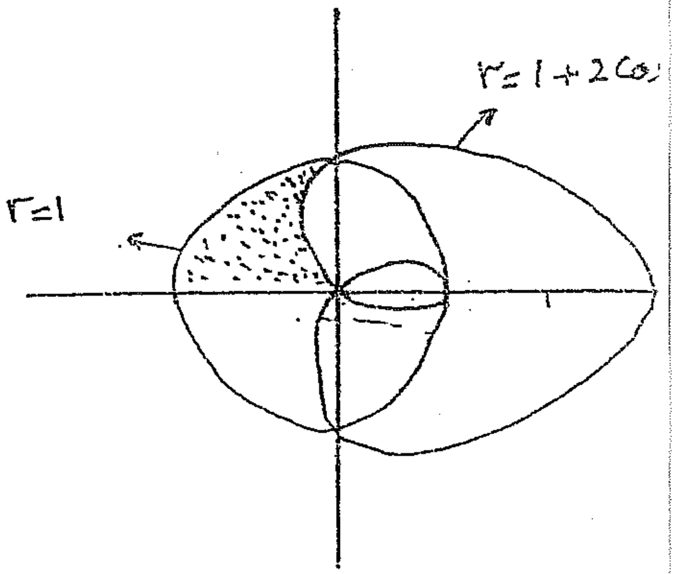
III (20 points): (a) Sketch the graph of $r = \dfrac{8}{2 - \sin\theta}$ and indicate the center, vertices, Foci and directrices.

(b) Use part (a) to sketch the graph of $r = \dfrac{8}{2 - \sin(\theta + \frac{\pi}{[illegible]})}$

IV (20 points): (a) Sketch the graph of the conic section $r = \dfrac{8}{2 - 4\cos\theta}$ indicating vertices foci and directrices in polar coordinates.

(b) Use (a) to sketch the graph of $r = \dfrac{8}{2 - 4\cos\left(\theta + \frac{\pi}{4}\right)}$ indicating vertices, foci and directrices in polar coordinates.

V (15 points): Find the shaded area.



IV (20 points): (a) Sketch the graph of the conic section $r = \dfrac{8}{2-4\cos\theta}$ indicating vertices foci and directrices in polar coordinates.

100

Birzeit University- Mathematics Department
Math 132

Dr. Marwan Aloqeili(Sec.2) and Dr. $\boxed{\text{Marwan Awartani (Sec.1\&3)}}$

Fall 2002/2003

Third Exam

Name:..Ali...Nayef.......Taqatqa.　　　Number:..1011273

There are 10 (T/F) questions and 14 multiple choice. **Calculators are not allowed.**

Question 1 Circle the most correct answer:

$\lim \frac{n}{n+10^6}$

70

1. $\sum_{n=1}^{\infty} \frac{1}{n+(10)^6}$
   (a) Converges to 0.
   (b) Converges by nth term test.
   (c) Diverges.
   (d) None.

2. One of the following p-series converges:
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$.
   (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$.
   (c) $\sum_{n=1}^{\infty} \frac{1}{n}$.
   (d) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$.

   $p > 1$

3. The sequence $a_n = (1 - \frac{1}{n^2})^n$
   (a) Diverges.
   (b) Converges to -1.
   (c) Converges to 1.
   (d) None of the above.

   $\lim_{n\to\infty} (1 - \frac{1}{n^2})^n$

   $= e^0 = 1$

4. $\sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1} n =$
   (a) $\pi/2$.
   (b) $\pi/4$.
   (c) $-\pi/4$.
   (d) None of the above.

   $\tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + \tan^{-1}(4) - \tan^{-1}(3)$

5. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$
   (a) Diverges by integral test.
   (b) Diverging by directly comparing it with $\sum \frac{1}{n}$.
   (c) Diverging by limit comparison test with $\sum \frac{1}{n}$.
   (d) All of the above.

$\frac{1}{2}\ln^2 b - \frac{1}{2}\ln^2 2$

$\frac{\ln n}{n} > \frac{1}{n}$

$\lim \frac{n \ln n}{n} = \infty$

$\int \frac{\ln n}{n} = \int \frac{u}{x} x\,du = \frac{u^2}{2}$

$\lim_{n\to\infty} (1 - \frac{1}{n^2})^n$

$\ln f(x) = \lim \frac{\ln(1 - \frac{1}{n^2})}{1/n}$

$= \frac{\frac{1}{1 - 1/n^2} \cdot \frac{2}{n^3}}{-1/n^2}$

$= \lim \frac{-1}{(1 - \frac{1}{n^2})n^3}$

$= \lim \frac{-1/n}{1 - \frac{1}{n^2}}$

$\lim \ln f(x) = 0$

$\Rightarrow \ln f(x) = 1$

$\Rightarrow \lim f(x) = e^0$

$(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + \dots + (\tan^{-1} n - \tan^{-1} n)$

$\frac{\pi}{2} - \frac{\pi}{4}$

$S_n = -\tan^{-1} 1 + \tan^{-1}(n+1) = \frac{\pi}{4}$

$$\frac{1+\cos^2 n}{n^2} < \frac{2}{n^2}$$

6. $\sum_{n=1}^{\infty} \frac{1+(\cos n)^2}{n^2}$

   (a) Converges.

   (b) Diverges.

   (c) Diverges by the nth term test.

   (d) Cannot determine.

7. $\sum_{n=1}^{\infty} e^{-n} =$ $\frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}$ $\left(\frac{1}{e}\right)^n$

   $\frac{1}{e^n} = \left(\frac{1}{e}\right)^n$

   $= \frac{1}{1-\frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$ as $\frac{1}{e}$ $r = \frac{1}{e}$

   (a) $\frac{e}{e-1}$.

   (b) $\frac{1}{e-1}$.

   (c) $\frac{1}{1-e}$.

   (d) None of the above.

   $\frac{1}{e(1-\frac{1}{e})} = \frac{1}{e-1}$

8. One of the following is true:

   (a) $\sum \frac{3^n}{n^3 2^n}$ diverges by ratio test.

   (b) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ converges by nth root test. ×

   (c) $\sum_{n=1}^{\infty} 2^n$ converges. ×

   (d) $\sum \frac{1}{\ln n}$ is a geometric series. ×

   $\lim \frac{3^{n+1}}{(n+1)^3 2^{n+1}} \cdot \frac{n^3 2^n}{3^n}$

   $= \lim \frac{n^3}{(n+1)^3}$

   $= \frac{3}{2}$

9. The nth partial sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is

   (a) $s_n = 1 - \frac{1}{\sqrt{2}}$.

   (b) $s_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$.

   (c) $s_n = 1 - \frac{1}{\sqrt{n+1}}$.

   (d) None of the above.

   $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$

   $1 - \frac{1}{\infty} = 1$

   $\left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

   $\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}\right) + \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}}\right)$

   $\cdots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

   $= 1 - \frac{1}{\sqrt{n+1}}$

10. If $a_1 = 2$, $a_{n+1} = \frac{2}{n} a_n$ then $\sum_{n=2}^{\infty} a_n$

    (a) Converges by ratio test.

    (b) Diverges.

    (c) Converges by integral test.

    (d) None of the above.

    $2 + 2(2) + 1(4) + ($

    $\lim \frac{a_{n+1}}{a_n}$

    $= \frac{\frac{2}{n} a_n}{a_n} = \frac{2}{n} = 0$

11. The series $\sum_{n=0}^{\infty} (\ln x)^n$ converges if

    (a) $-1 < x < 1$.

    (b) $-e < x < e$. ×

    (c) $e^{-1} < x < e$.

    (d) None.

$\sum_{n=1}^{\infty} (\ln x)^{n-1}$

$a_1 = (\ln x)^0 = 1$

$a_2 = (\ln x)$

$a_3 = (\ln x)^2$ $\Rightarrow r = (\ln x)$

$S_n = \frac{1}{1-(\ln x)}$

$e^{-1} < e^{\ln x} < e^1$

12. One of the following series converges:

(a) $\sum_{n=1}^{\infty} n^2$.

(b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(d) $\sum_{n=1}^{\infty} \ln(1/n)$.

$-5 \sum$

13. $\sum_{n=1}^{\infty} \frac{-5}{n}$

(a) Converges to 0.

(b) Converges to 1.

(c) Diverges.

(d) None of the above.

$\lim \frac{2^{\cancel{n+1}}}{(n+1)^2} \frac{n^2}{\cancel{2^n}}$

$= 2\lim \frac{n^2}{n^2+2n+1}$

$= 2 > 1 \Rightarrow div.$

$\frac{(-1)^{n+2}}{(-1)^{n+1}}$ $(-1)^{n+2-}$ (-1)

14. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(a) Converges.

(b) Diverges.

(c) Converges to 0.

(d) None.

Question 2. Answer by True or False:

30

1. The series $\sum_{n=1}^{\infty}(-1)^{n+1}$ diverges. (.true)
2. If $\lim_{n\to\infty}(a_n)^{1/n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges. (.false)
3. Any increasing sequence and bounded from above converges . (.true)
4. $\lim_{n\to\infty} \frac{x^n}{n!} = 0$. (.true)
5. The sequence $a_n = 1 + (-1)^n$ diverges. (.true)
6. If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges. (.false)
7. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is called the harmonic series. (.false)
8. The sequence $a_n = \frac{3n+1}{n+1}$ is nondecreasing. (.true)
9. The series $\sum_{n=1}^{\infty}(\sin x)^n$ converges for any value of x. (..false)
10. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} ka_n$, k is any constant, converges. (.true)

$\sum_{n=2} (-1)^n$ $\frac{(-1)^n}{(-1)}$

$\lim_{n\to\infty} \frac{(-1)^{n+2}}{(-1)^{n+1}} = -1$

0, 1, 0, 1

$\sum_{n=2} (-1)^n$

$\frac{3(n+1) - (3n}{(n+1)^2}$

$\frac{3n+3-}{(n+1)^2}$

$= \frac{2}{}$

Birzeit University
Math. & Comp. Science Dept.
Math. 132

Second Semester

Third Hour Exam

The Name of Discussion Teacher:

Student Name:________________________________**Number:**__________ **Section**____

I (40 points): Circle the MOST correct answer:

1. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

   (a) $y = -\frac{1}{8}$
   (b) $y = \frac{7}{8}$
   (c) $y = -\frac{9}{8}$
   (d) $y = \frac{9}{8}$

2. A circle of radius a and an ellipse of major semiaxis a both centered at the origin meets in

   (a) 4 points
   (b) 1 points
   (c) 2 point
   (d) 0 point

3. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

   (a) Parabola
   (b) Ellipse
   (c) Hyperbola
   (d) Circle

4. A parametrization of the line segment with initial point $(0,1)$ and terminal point $(1,0)$ is

   (a) $x = \sin^2 t \quad , y = \cos^2 t , \ 0 \le t \le \frac{\pi}{2}$
   (b) $x = 1 - t \quad , y = t \quad , 0 \le t \le 1$
   (c) $x = t \quad , y = 1 + t \quad , -1 \le t \le 0$
   (d) $x = 1 + t \quad , y = t \quad , 0 \le t \le 1$

5. The Parametrization $x(t) = -\cos t \ , \ y(t) = \sin t \ , \ 0 \leq t \leq 2\pi$

(a) A circle with initial point $(1, 0)$ counterclockwise.
(b) A circle with initial point $(-1, 0)$ counterclockwise.
(c) A circle with initial point $(-1, 0)$ clockwise.
(d) A circle with initial point $(1, 0)$ clockwise.

6. The length of the curve $x(t) = \sin t \ , \ y(t) = 1 + \cos t \ , \ 0 \leq t \leq \frac{\pi}{2}$ is:

(a) π
(b) 2π
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$

7. The slope of the tangent line to the curve $y(t) = t^2 - \sin t \ , \ x(t) = \cos t$, at $t = \dfrac{\pi}{2}$ is:

(a) $\frac{\pi^2}{2}$
(b) $\frac{-1}{\pi}$
(c) -2
(d) $-\pi$

8. Which one of the following points lies on the curve $r = \cos 2\theta$

(a) $(0, 0)$
(b) $(1, \frac{\pi}{2})$
(c) $(\frac{1}{2}, \frac{\pi}{3})$
(d) all

9. The graph of $r = 2 \csc \theta$ is

(a) Circle

(c) Hyperbola
(d) Parabola

10. The curves $\theta = \frac{\pi}{2}$ and $r = 0$

(a) never meets.
(b) intersect in one point.
(c) intersect in infinitely many points.
(d) are identical.

11. $7^{\log_7^5} =$

(a) $\frac{5}{7}$
(b) $\frac{7}{5}$
(c) 5
(d) 7

12. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

(a) $y = -\dfrac{1}{8}$
(b) $y = \dfrac{7}{8}$
(c) $y = -\dfrac{9}{8}$
(d) $y = \dfrac{9}{8}$

13. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

(a) Parabola
(b) Ellipse
(c) Hyperbola
(d) Circle

14. A parametrization of the line segment with initial point (0,1) and terminal point (1,0) is

(a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$
(b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$
(c) $x = 1$, $y = 1 + t$, $-1 \leq t \leq 0$
(d) $x = 1 + t$, $y = 1$, $0 \leq t \leq 1$

15. $\dfrac{\log_2(x)}{\log_8(x)} =$

(a) $\dfrac{2}{8}$
(b) $\dfrac{1}{3}$
(c) $\log_4^{x}$
(d) 3

16. $\sec^{-1}(-2)$

(a) $\dfrac{2\pi}{3}$
(b) $-\dfrac{\pi}{3}$
(c) $-\dfrac{2\pi}{3}$
(d) $\dfrac{3\pi}{[illegible]}$

17. $\lim_{x \to 0^+} (\ln x - \ln(\sin x))$

(a) 0

(b) -1

(c) $+1$

(d) Doesn't exist.

18. $\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$

(a) ∞

(b) 0

(c) 1

(d) Doesn't exist

19. $\lim_{x \to \frac{\pi^-}{2}} \dfrac{\sec x}{\tan x} =$

(a) 0

(b) 1

(c) ∞

(d) Doesn't exist

20. The order of the functions $x^2, e^x, \ln x$ from slowest growing to fasters growing as $x \to 0$ is

(a) e^x , $\ln x$, [illegible]

(b) e^x , x^2 , $\ln x$

(c) x^2 , e^x , $\ln x$

(d) $\ln x$, x^2 , e^x

II (20 points): Suppose the path of a moving particle in a plane is described by

$$\begin{array}{l} x(t) = 3 + 4\sin(t) \\ y(t) = 2 + 5\cos(t) \end{array}, \quad 0 \le t \le \pi$$

1. Sketch the path of motion and determine the direction.
2. Find the equation of the tangent line at $t = \frac{\pi}{3}$.

Question #4:

Graph the conic section $2x^2+3xy+2y^2-1=0$ in the xy-plane indicating the center, the vertices and the foci in the xy coordinates.

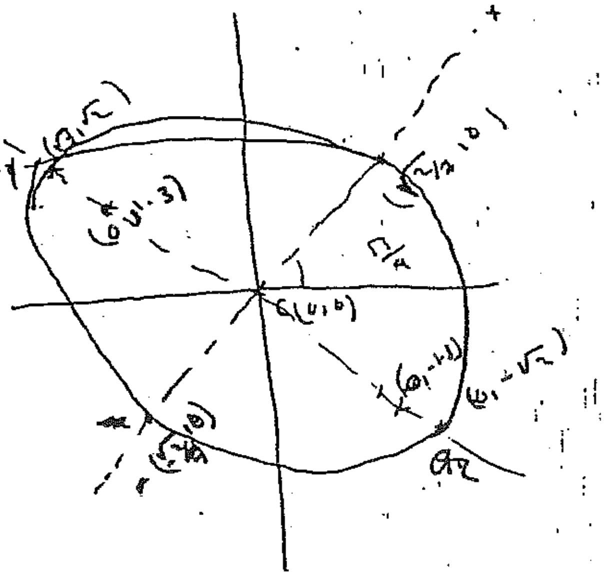
$2x^2+3xy+2y^2-1=0$

$B^2-4AC = 9-4(2)(2) < 0$ ellips

$\cot 2\alpha = \frac{A-C}{B} = \frac{2-2}{3} = 0$

$2\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{4}$



$x = x'\cos\frac{\pi}{4} - y'\cos\frac{\pi}{4}$

$x = \frac{x'}{\sqrt2} - \frac{y'}{\sqrt2}$

$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$

$= \frac{x'}{\sqrt2} + \frac{y'}{\sqrt2}$

Equation became

$2\left(\frac{x'-y'}{\sqrt2}\right)^2 + 3\left(\frac{x'}{\sqrt2}-\frac{y'}{\sqrt2}\right)\left(\frac{x'}{\sqrt2}+\frac{y'}{\sqrt2}\right) + 2\left(\frac{x'+y'}{\sqrt2}\right)^2 - 1 = 0$

$\cancel{2}\frac{(x'^2+y'^2-2x'y')}{\cancel{2}} + 3\left(\frac{x'^2-y'^2}{2}\right) + \cancel{2}\left(\frac{x'^2+y'^2+2x'y'}{\cancel{2}}\right) - 1 = 0$

$x'^2+y'^2+\frac{3x'^2}{2}-\frac{3y'^2}{2}+x'^2+y'^2 = 1$

$\frac{7}{2}x'^2+\frac{1}{2}y' = 1$ $\qquad \frac{x'^2}{2/7}+\frac{y'}{2} = 1$

Foci $\qquad c = \sqrt{a^2-b^2}$

$= \sqrt{2-\frac{2}{7}} = \sqrt{1.714}$

$= 1.3$

Foci at x'y' plan $= (0, \mp 1.3)$

at xy plan $\quad y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$

$= 0 + 1.3(\cos\pi/4)$

$= \frac{1.3}{\sqrt2}$

$x = x'\cos\frac{\pi}{4} - y'\cos\frac{\pi}{4}$

Foci in xy plan =

Foci $\left(\frac{1.3}{\sqrt2}, -\frac{1.3}{\sqrt2}\right)$ $\left(\frac{1.3}{\sqrt2}, \frac{1.3}{\sqrt2}\right)$

~~[illegible]~~

vertics $\quad a = (0, \mp\sqrt{2})$.

$x = \cancel{x'}\cancel{\cos x} - y'\sin x$

$x = 0 - \frac{\sqrt{2}}{\sqrt{2}} = -1$

$\boxed{3}$ Vertics in xy plang

~~$a_1(\ldots)$~~

$a_1 = (-1, 1)$

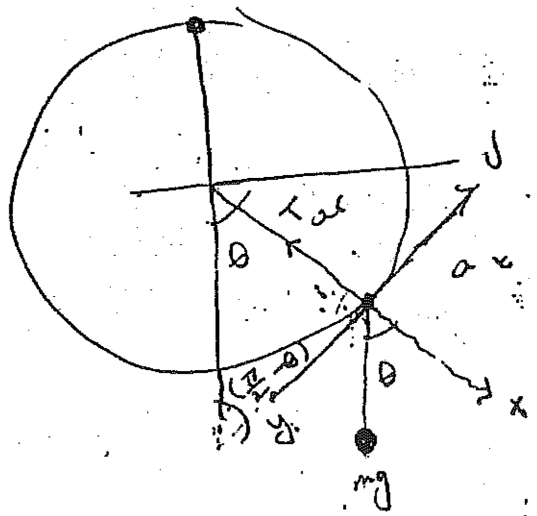
$a_2 = (1, -1)$

$y = \underbrace{x'\sin x}_{0} + y'\cos x$

$= \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\frac{mv^2}{R} = \underbrace{mg}_{T=0}$

$\boxed{v = \sqrt{Rg}}$



$\frac{mv^2}{R}$

$\boxed{mg\cos\theta + \frac{mv^2}{R} = T}$ (1)

$\boxed{a_T = -mg\sin\theta}$

Q#7: Find the series radius of convergence. For what values of x does the series converge :

a) Conditionally

b) absolutely

(i) $\frac{(x-2)^n}{n}$ (ii) $\frac{(-1)^n x^n}{n!}$

Q#8:

a) Find the maclurin series for $f(x) = \ln(1 + x^2)$.

b) How many terms are supposed to be used to get an estimate f(0.2) with error less that 10^{-6}.

#9: **Evaluate:** $\sum_{n=1}^{\infty} nx^n$ if $|x| < 1$

MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min. **JANUARY 2008**

NAME: George Hannuneh

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Jad'on

82

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]

CIRCLE THE RIGHT ANSWER:

28

30

10

14

82

1. Let $f(x)=\sum_{n=0}^{\infty}x^n$. The interval of convergence of the definite integral 0 to x, $\int_0^x f(t)dt$ is

$\frac{x^{n+1}}{n+1}\Big|_0 = \frac{x^{x+1}}{x+1} - x$?

$\frac{x^{n+1} - x(x+1)}{x+1}$

$\ln|1+x|$

(A) $x = 0$ only
(B) $|x| \le 1$
(C) $-\infty < x < \infty$
(D) ~~$-1 \le x < 1$~~
(E) $-1 < x < 1$

$-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}$

2. The coefficient of x^4 in the Maclaurin series for $f(x)=e^{-x/2}$ is

(A) -1/24
(B) 1/24
(C) 1/96
(D) -1/384
(E) 1/384

$\frac{1}{e^{\frac{x}{2}}}$ $-\frac{e^{-\frac{x}{2}}}{2}$ $-\frac{1}{2e^{\frac{x}{2}}}$

$\frac{(-1)^n (x)^n}{2^{n+1} n!}$ $\frac{e^{-\frac{x}{2}}}{4}$

$\frac{1}{32}\frac{x^4}{4\times3\times2}$ $\frac{1}{4e^{\frac{x}{2}}}$

3. Which of the following series diverges?

(A) $\Sigma 1/n^2$ conv
(B) $\Sigma 1/(n^2+n)$ conv.
(C) $\Sigma n/(n^3+1)$
(D) $\sum \frac{n}{\sqrt{(4n^2-1)}}$ $\frac{n}{n\sqrt{4-\frac{1}{n^2}}}$ $\frac{1}{4}$
(E) none of the preceding.

4. For which of the following series does the Ratio Test fail?
   (A) $\Sigma 1/n!$
   (B) $\Sigma n/2^n$
   (C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \ldots$
   (D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \ldots$
   (E) $\Sigma n^n/n!$

5. Which of the following alternating series diverges?
   (A) $\Sigma\, (-1)^{n-1}/n$
   (B) $\Sigma\, (-1)^{n+1}(n-1)/(n+1)$
   (C) $\Sigma\, (-1)^{n+1}/\ln(n+1)$
   (D) $\sum \dfrac{(-1)^{n-1}}{\sqrt{n}}$
   (E) $\Sigma\, (-1)^{n-1} n/n^2 + 1$

6. Which of the following series converges conditionally?
   (A) $3 - 1 + 1/9 - 1/27 + \ldots$
   (B) $\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} - \ldots\ldots\ldots\ldots$
   (C) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$
   (D) $1 - 1.1 + 1.21 - 1.332 + \ldots$
   (E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \ldots$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \Re$.

   If $f(0) = 2, \quad f'(0) = 2$ *and* $f^{n}(0) = 3$ for $\quad n \geq 2$ then $f(x) =$

   (A) $3e^{x} + 2x - 1$

   (B) $e^{3x} + 2x + 1$

   (C) $e^{3x} - x + 1$

   (D) $3e^{x} - x - 1$

   (E) $3e^{x} + 5x + 5$

$\frac{\ln\left(\frac{1}{n^3}\right)}{\frac{1}{n^3}}$

n^3

∞

8. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(A) I only (B) II only (C) III only

(D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

$\frac{x^n}{n!}$

(A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!\,e}$

(D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

$\frac{e(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$\frac{1}{2}$ $\frac{a_{n+1}}{a_n} \le \frac{n+4}{2n+1}$ for all n. Then $\lim_{n\to\infty} a_n =$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \le \frac{1}{2}$

$\Rightarrow \sum a_n$ converges

$\Rightarrow \lim_{n\to\infty} a_n = 0$.

(A) 0 (B) 1/2 (C) 1

(D) 2 (E) 4

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ $\Rightarrow$ Converges by Integral test

$\int_3^\infty \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99}\Big|_3^\infty$

$= \frac{-1}{99(\ln n)^{99}}\Big|_3^\infty = \frac{-1}{\infty} + \frac{1}{99(\ln 3)^{99}}$ $\Rightarrow$ Integral Converges

(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$ ⇒ Converges by the nth Root test absol.

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n\to\infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

(c) $\sum_{n=1}^{\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3}$ diverges by limit comparison test

$$\lim_{n\to\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3} \div \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{n^4-n^{\frac{3}{2}}+6n}{n^4-n-3}$$

$= 1 \Rightarrow$ ~~but~~ both diverge or converg

$\frac{1}{n}$ diverges. (power series with $p=1$)

⇒ both diverg

30

QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

(a) When $x = -4$ does this series converge or diverge?
(b) Determine all values for which the series converges.

$$\lim_{n \to \infty} \sqrt[n]{\frac{(-2)^n}{n^{\frac{1}{2}}} (x+3)^n}$$

$$= \frac{-2}{n^{\frac{1}{2n}}} (x+3) \quad \therefore \quad |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-\frac{1}{2} < x+3 < \frac{1}{2}$$

$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n \to \infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n \to \infty} 2^n$$

$$= \infty$$

both diverg by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

$\Rightarrow$ the series converges on the interval $(-4, -2]$

10

QUESTION FOUR: [16 points]

Consider the integral $\int x\cos(x^3)dx$.

(a) Write down the Maclaurin series for cos(x), $\cos(x^3)$, and $x\cos(x^3)$.

(b) Evaluate $\int_0^1 x\cos(x^3)dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{\cancel{3}6n!}$$

Maclaurin

$$\cos(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!}$$

$$x\cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x\cos(x^3) = x + \frac{x^7}{2!} + \cancel{\frac{x^{13}}{4!}} + \cancel{\frac{x^{19}}{6!}} + \cancel{\frac{x^{25}}{8!}} + \cancel{R}$$

$$\int_0^1 x\cos(x^3) = \frac{2x + x^2}{2}\Big|_0^1 \quad \cancel{\text{where } R \le a_5}$$

$$\int_0^1 x\cos(x^3) = \boxed{\tfrac{3}{2}} \; ?$$

14

Key

Birzeit University
Department of Mathematics
Math 132

Final exam **Summer/2009**
Name : .... **Number:...**
Instructor:............ **Section :......**

Q#1 (72%) Circle the correct answer.

$\ln \frac{\pi}{3} \times \frac{6}{\pi}$

1) $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1+x^2)\tan^{-1}x} =$

a) $\ln 4 - \ln 3$

b) $\ln 2$

c) $\ln 3 - \ln 2$

d) $\frac{\pi}{12}$

2) If $y = (\ln x)^x$ then $\frac{dy}{dx}$

a) $(\ln x)^x \left(\frac{1}{\ln x} + \ln x\right)$

b) $(\ln x)^{x-1}$

c) $x(\ln x)^{x-1}$

d) $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$

3) $\displaystyle\int_1^{e^3} \frac{1}{x\sqrt{(1+\ln x)}}dx =$

a) $\ln \frac{4}{3}$ b) 2 c) $\frac{4}{3}$ d) $\ln \frac{3}{2}$

4) $\displaystyle\int_1^2 \frac{\sinh(\ln x)}{x}dx =$

a) 0

b) 1

c) $\frac{1}{4}$

d) $\frac{1}{2}$

5) if $y=\tan^{-1}(\frac{1}{x})$ then $\csc y =$

a) $\frac{\sqrt{1+x^2}}{x}$ b) $\sqrt{1+x^2}$ c) $\frac{x}{\sqrt{1+x^2}}$ d) $\frac{1}{\sqrt{1+x^2}}$

6) if $y=5^{\ln x}$ then $\frac{dy}{dx}$ when x=1 is:

a) 0 b) $-\ln 5$ c) $\ln 5$ d) 1

7) $\int_0^1 e^{\sqrt{x}}\, dx =$

a) 0 b) 4 c) 3 d) 2

8) $\int x^2 e^{3x}\, dx =$

a) $\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c$

b) $\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}-\frac{2}{27}e^{3x}+c$

c) $\frac{1}{3}x^2e^{3x}-\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c$

d) $-\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}-\frac{2}{27}e^{3x}+c$

9) $\int \frac{dx}{(x+2)\sqrt{x^2+4x}} =$

a) $\sqrt{x^2+4x}+c$

b) $\frac{1}{2}\sec^{-1}\left|\frac{x+2}{2}\right|+c$

c) $\frac{1}{2}\ln\left|x^2+2x\right|+c$

d) $\sinh^{-1}(x+2)+c$

10) If $f(x) = xe^x$, then $(f^{-1})'(e) =$

(a) $\dfrac{1}{e^e(e+1)}$ (b) $e^e + ee^e$ (c) $\dfrac{1}{2e}$ (d)None of the above

$xe^x + e^x$

11) Consider the improper integrals:

(i) $\displaystyle\int_3^{\infty} \frac{dx}{(x-3)^2}$ $\left.\frac{-1}{x-3}\right|_3$ (ii) $\displaystyle\int_{-5}^{\infty} \frac{dx}{\sqrt{x+5}}$

(a) only integral (i) converge b)only integral (ii) converges

(b) both integrals converge d)both integrals diverge.

12. The power series $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \cdots \cdots + \dfrac{x^n}{n} + \cdots$ converges if and only if

$\dfrac{x}{1-x}$ $1 + x + x^2 + \cdots + x^{n-1}$

(a) $-1 < x < 1$ (b) $-1 \le x \le 1$

(c) $-1 \le x < 1$ (d) $-1 < x \le 1$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

13. The Maclaurin series of order 3 for $f(x) = \sqrt{x+1}$ is

(a) $1 + \dfrac{x}{2} - \dfrac{x^2}{4} + \dfrac{3x^3}{8}$ (b) $1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16}$

(c) $1 - \dfrac{x}{2} + \dfrac{x^2}{8} - \dfrac{x^3}{16}$ (d) $1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{8}$

??

14) Determine whether $\displaystyle\int_2^{30} \frac{dx}{(x-3)^{2/3}}$ converges or diverges. If the integral converges, find its value

a)converges, 15 b)converges, 3

c)converges, 1 d)diverges.

$\displaystyle\int_2^3 + \int_3^{30}$

$\left.3\sqrt[3]{x-3}\,\right|_2^3 + \left.3\sqrt[3]{x-3}\,\right|_3^{30}$ 3

$0 - 3\sqrt[3]{-1}$ $9 - 0$

3

15) If $(\frac{1+i}{1-i})^4 + z = 2+i$ then z=

a) 2-i

b) 1-2i

c) 2+4i

d) 1+i

16) The series $\sum_1^\infty \frac{n^n}{2^n 3^n}$

a) Diverges by Ratio test

b) Converges by n^{th} term test

c) Converges by Integral test

d) Converges by n^{th} root test

17) The series $\sum_1^\infty (-1)^n (\frac{n^2}{2n^4+5})$

a) Converges by limit comparison test

b) Diverges by n^{th} term test

c) Converges by n^{th} term test

d) Converges absolutely

18) The sequence $\{a_n\} = \{\frac{1+(-1)^n}{n}\}$

a) Converges to 1

b) Converges to 0

c) Converges to 2

d) Diverges

19) $\int \sin^{-1} x \, dx =$

a) $x \sin^{-1} x - 2\sqrt{1-x^2} + c$

b) $x \sin^{-1} x + 2\sqrt{1-x^2} + c$

c) $x \sin^{-1} x - \sqrt{1-x^2} + c$

d) $x \sin^{-1} x + \sqrt{1-x^2} + c$

$u = \sin^{-1} x$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} dx$ $\quad v = x$

$x \sin^{-1} x + \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

20) $\int \frac{dx}{\sqrt{2x - x^2}} =$

$a) 2\sqrt{2x - x^2} + c$

$b) \sin^{-1}(x - 1) + c$

$c) \sin^{-1}(x - 2) + c$

$d) \sec^{-1}(x - 1) + c$

21) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{(n+1)(n+2)}}$

a) Converge conditionally

b) Converge absolutely

c) Diverges

d) Converges to 2

22) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\frac{1}{\ln n} > \frac{1}{n}$ ✓

$a)$ Converges Absolutely

$b)$ Converges conditionally

$c)$ Divergess by alternating series theorem

$d)$ Diverges by n^{th}-term test

$e^x = e^1$

23) If cosh x + sinhx=e then x=

a) e

b) 1

$c) \ln 5$

$d) \ln \sqrt{5}$

24) $\int_3^4 \frac{3dx}{x^2 + x - 2} =$

$a) \ln \frac{5}{4}$ $b) \ln \frac{4}{5}$ $c) \ln \frac{8}{5}$ $d) \ln \frac{5}{8}$

$\sum \frac{1}{x-1} - \frac{1}{x+2}$

$\ln \frac{x-1}{x+2} \Big|_3^4 = \ln \frac{\cancel{3}}{\cancel{6}_{2}} \times \frac{5}{2}$

Q2(9%) Use series to find an estimate for $\int_0^{\frac{1}{2}} \frac{\tan^{-1} x}{x} dx$ with an error of magnitude less than 10^{-3}

Q3(10%) Solve $z^4 = -81$ in the field of complex numbers

Q4)(10%) Sketch the graph of the conic section

$$9x^2 + 25y^2 + 18x - 100y = 116$$ indicating

a)foci ....

b) vertices ....

c)eccentricity...

d) directrices..

Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Marwan Awartani

Fall

Final Exam

Student Name:______________________ Number:__________ Section: ____

Q1: (60 points) Circle the MOST correct answer:

1. $\int_e^{e^2} = \frac{1}{x\ln x}dx =$

   $u = \ln x$
   $du = \frac{dx}{x} \quad dx = x\,du$
   $= \int_e^{e^2} \frac{du}{u} = \ln|u|\Big]_e^{e^2} = \ln|\ln x|\Big]_e^{e^2}$
   $= \ln \ln e^2 - \ln \ln e$
   $= \ln 2$

   (a) 1
   (b) ln2
   (c) $\ln\left(\frac{1}{2}\right)$
   (d) 0

2. The curve with parametric equations $x = \sin t$, [illegible] $-\infty < t < \infty$.

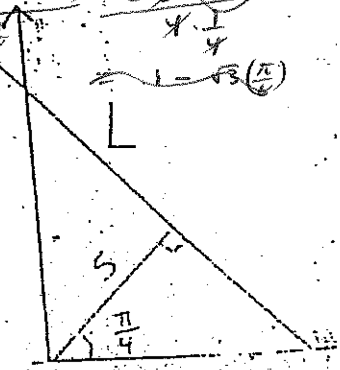
   (a) A segment of a parabola
   (b) A circle
   (c) An ellipse
   (d) A hyperbola

3. The slope of the curve $x = \sin 2t, y = \cos t$ at $t = +\frac{\pi}{6}$ is:

   $x = 2\sin t \cos t$
   $x = 2\sin\frac{\pi}{6}\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$
   $x = 2\sin t\, y$
   $y = \frac{x}{2\sin t} \Rightarrow y' = \frac{2\sin t - 2\cos t(x)}{4\sin^2 t} = \frac{2\cdot\frac{1}{2} - 2\cdot\frac{\sqrt{3}}{2}\left(\frac{\pi}{6}\right)}{4\cdot\frac{1}{4}}$
   $= 1 - \sqrt{3}\left(\frac{\pi}{6}\right)$

   (a) 0
   (b) 1
   (c) $\frac{1}{2}$
   (d) $-\frac{1}{2}$

4. The polar equation of straight line L is:

   (a) $r\cos\left(\theta + \frac{\pi}{4}\right) = -5$
   (b) $r\cos\left(\theta - \frac{\pi}{4}\right) = 5$
   (c) $r\sin\left(\theta + \frac{\pi}{4}\right) = -5$
   (d) $r\cos\theta = 5$

5. The graphs of the curves with polar coordinates $r = \sin\theta$, $r = -\cos\theta$ intersects at:

(a) Only at the origin.

(b) Only when $\tan\theta = -1$

(c) At exactly two points.

(d) At exactly three points.

? 6. The equation of $x^2 + 5xy + y^2 = 3$ is

(a) Circle

(b) Ellipse

(c) A hyperbola ×

(d) A parabola

7. One of the following is not an improper integral

(a) $\int_0^{10} \frac{\sin x}{x} dx$ $= \int (x - \frac{x^3}{3!}$ ...

(b) $\int_{-\infty}^{a} x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_0^\infty \frac{dx}{x^2 - 1}$

$du = \cos x \, dx$
$u = \sin x$

8. $\int \sin^2 x \cos^3 x dx =$

$= \int \sin^2 x \cos^2 x \cos x \, dx$
$= \int u^2 (1-u^2) \, du = \int (u^2 - u^4) \, du$
$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{15} + c$

(d) $\frac{\sin^3 x}{15}(5 + 3\sin^2 x) + c$

9. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1 - t^4}$, $-1 \le t \le 1$, then

(a) The initial point is (1,0) and the end point is (0,1).

(b) the motion is clockwise.

(c) the motion is counter clockwise.

(d) None of the above.

? 10. $\lim_{x \to 0^+} (1 + x)^{\frac{1}{x}}$

$u = \frac{1}{x}$
$x \to 0 \Rightarrow u \to \infty$

(a) e

(b) 1

(c) $\frac{1}{e}$

(d) 0

$\lim_{u\to\infty} (1 + \frac{1}{u})^u$

10. The graph of the curves with polar coordinates $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$ intersects in

(a) One point only
(b) Two points only
(c) Three points only
(d) Four points only

11. The slope of the polar curve $r = 1 + 2\cos\theta$ at the origin is

(a) 1
(b) -1
(c) $\sqrt{3}$
(d) $\pm\sqrt{3}$

? 12. $\lim_{x \to 0^+} (1 + x)^{\frac{1}{x}} =$

(a) e
(b) 1
(c) $\frac{1}{e}$
(d) 0

? 13. $\lim_{x \to 0} \frac{\tan^{-1} x}{x} =$

(a) 1
(b) -1
(c) 0
(d) ∞

14. $7^{1-[illegible]} =$

(a) [illegible]
(b) $\frac{7}{5}$
(c) 5
(d) 7

15. The eccentricity of the conic section $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

(a) $\frac{5}{3}$
(b) $\frac{5}{4}$
(c) $\frac{3}{4}$
(d) $\frac{4}{3}$

16. The length of the polar curve $r = 4\cos\theta$, $0 \le \theta \le \frac{\pi}{2}$ is

(a) π
(b) 2π
(c) 3π

$u = \tan^{-1} x$, $x = \tan u$, $dx = \sec^2 u \cdot du$; $x = 0$, $u = 0$

$\lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{u \to 0} \frac{u}{\tan u} = \lim_{u \to 0} \frac{u \cos u}{\sin u} =$

$x \to 0$, $u \to 0$; $x = \tan u$

$u = \tan^{-1} x$, $du = \frac{dx}{1+x^2}$, $dx = (1+x^2) \cdot du$

$= \lim_{x \to 0} \frac{(1+x^2)}{x} \cdot du = \frac{1 + \tan^2 u}{\tan u} \cdot du$

$= \int \frac{\sec^2 u}{\tan u} \cdot du$

$= \int \frac{1}{\cos^2 u} \cdot \frac{\cos u}{\sin u} \cdot du = \int$

$e = \frac{c}{a} = \frac{5}{3}$

$a = 3$, $b = 4$

$c^2 = a^2 + b^2 = 9 + 16$, $c = 5$

17. The surface area generated by revolving $r = 4\cos\theta \;\; 0 \le \theta \le \frac{\pi}{2}$ about x-axis is

(a) 2π
(b) 4π
(c) 8π
(d) 16π

18. The Cartesian coordinates of the point $P(-4, \frac{\pi}{4})$ is

(a) $(-2\sqrt{2}, 2\sqrt{2})$
(b) $(-2\sqrt{2}, -2\sqrt{2})$
(c) $(2\sqrt{2}, 2\sqrt{2})$
(d) $(2\sqrt{2}, -2\sqrt{2})$

19. The angle θ that eliminate xy term in $2x^2 + \sqrt{3}xy + y^2 - 2y = 6$ is

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

20. $\sec^{-1}(-\sqrt{2}) =$ $\quad -\sqrt{2} = \sec y$

21. $\frac{\pi}{4}$ $\quad -\sqrt{2} = \frac{1}{\cos y}$

22. $\frac{-\pi}{1}$

23. $\frac{3\pi}{4}$ $\quad \cos y = \frac{-1}{\sqrt{2}}$ $\quad \frac{\pi}{4} - \frac{\pi 4}{4} = -\frac{3\pi}{4}$

24. $\frac{5\pi}{4}$ $\quad y = \frac{\pi}{4} + \frac{\pi 4}{4} = \frac{5\pi}{4}$

II (15 points): Evaluate the following integrals

$\int \sin^{\frac{5}{2}} x$.

Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Hasan Yousef

Final Exam **Summer**

Student Name:____________________________**Number:**__________ **Section**____

I (40points) : Circle the MOST correct answer:

1. If $\sinh x = \frac{-3}{4}$ then $\cosh x =$

   4   -3 5

   (a) $\frac{3}{5}$
   (b) $\frac{5}{4}$
   (c) $\frac{-3}{5}$
   (d) $\frac{-5}{4}$

2. The conic section with Foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$ is an

   $c = 1$ $a = 2$
   $a > c$
   $e < 1$

   (a) Ellipse
   (b) Parabola
   (c) Hyperbola
   (d) A circle

3. The conic section with eccentricity $\frac{1}{2}$ and directrix $x = 2$ has equation

   (a) $2x^2 + y^2 = 1$
   (b) $x^2 - y^2 = 1$
   (c) $y^2 - x^2 = 2$
   (d) $x^2 + \frac{4}{3}y^2 = 1$

   $e = \frac{c}{a} = \frac{1}{2}$ $2 \cdot \frac{1}{2}$ $\frac{a}{e} = 2$

   $\frac{a}{2} = \frac{c}{a} = \frac{1}{2}$ $e = \frac{a}{2}$

   $a^2 = 2c$ $\boxed{a = 1}$ $2c = a$

   $a = 2e$
   $= 2 \cdot \frac{1}{2}$
   $= 1$
   $c = ea = \frac{1}{2} \cdot 1$
   $\boxed{c = \frac{1}{2}}$

   $a = 1$

   $D = \frac{a}{e} = 2$
   $\boxed{a = 2e}$
   $e = \frac{c}{a} = \frac{1}{2}$
   $\boxed{a = 1}$
   $a = 2c \rightarrow \boxed{c = \frac{1}{2}}$
   $b^2 = a^2 - c^2$
   $= 1 - \frac{1}{4}$
   $\boxed{b = \frac{\sqrt{3}}{2}}$ $= \frac{3}{4}$

4. The directric of the parabola $x = \frac{y^2}{2}$ is given by

   (a) $x = 1$
   (b) $y = \frac{-1}{2}$
   (c) $y = \frac{1}{2}$
   (d) $x = \frac{1}{2}$

   $\frac{x^2}{1} + \frac{4y^2}{3} = 1$

   $y^2 = 2x$
   $4px = 2x$
   $\boxed{p = \frac{1}{2}}$

? 5. The conic section $x^2 + 4xy + \sqrt{2}y^2 + 5 = 0$ is

(a) Ellips

(b) Parabola

(c) Hyperbola

(d) A Circle

$\frac{x^2 + \sqrt{2}y^2}{-4xy-5} = \frac{-4xy-5}{-4xy-5}$

$= \frac{x^2}{-4y} + \frac{\sqrt{2}y}{-4x} = \frac{5}{-4xy}$

~~$-4x^2 - 4\sqrt{2}y^2 = \frac{5}{-4xy}$~~

? 6. $x = \cos 2t,\ \ y = \sin^2 t\ ,\ 0 \le t \le \frac{\pi}{4}$ represents

(a) half of a circle

(b) half of an Ellipse

(c) a line seqment

(d) a parabola

? 7. The slope of the Ellipse $x = 2\sin t,\ y = 3\cos t \quad 0 \le t \le 2\pi$ at the point $\left(\sqrt{2}, \frac{3}{\sqrt{2}}\right)$ is

(a) $-\frac{2}{3}$

(b) 3

(c) $\frac{3}{2}$

(d) $\frac{-3}{2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{4\sin^2 t}{a^2} + \frac{9\cos^2 t}{b^2} = 1$

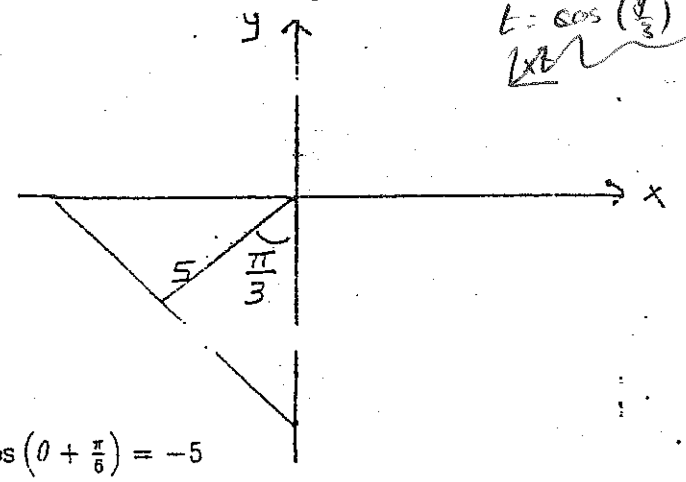
$t = \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{4}$

$t = \cos^{-1}\left(\frac{y}{3}\right) = \frac{\pi}{4}$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

8. An equation of the line in the figure is



(a) $r\cos\left(\theta + \frac{\pi}{6}\right) = -5$

(b) $r\sin\left(\theta + \frac{2\pi}{3}\right) = -5$

(c) $r\cos\left(\theta + \frac{5\pi}{6}\right) = 5$

(d) $r\sin\left(\theta + \frac{\pi}{6}\right) = -5$

9. The polar Equation of the circle with center $P(-2, \frac{\pi}{4})$ and radius 4 is

(a) $r = -4\sin\left(\theta + \frac{\pi}{4}\right)$

(b) $r = -4\sin\left(\theta + \frac{3\pi}{4}\right)$

(c) $r = 4\sin\left(\theta - \frac{3\pi}{4}\right)$

(d) $r = 4\sin\left(\theta - \frac{\pi}{2}\right)$

$$= \int \frac{x^{\cancel{2}} \cdot du}{2\cancel{x}\sqrt{1+x^2}}$$

$u = x^2$

$du = 2x \cdot dx$

$dx = \frac{du}{2x}$

$$= \int \frac{x \cdot du}{2\sqrt{1+x^2}} = \int \frac{\sqrt{u} \cdot du}{2\sqrt{1+u}}$$

$u = 1+x^2$

$du = 2x \cdot dx$

$dx = \frac{du}{2x}$

Q2:(15 points) (a) $\int \frac{x^2 dx}{\sqrt{1+x^2}} =$

$$= \cancel{\int \frac{(u-1) \cdot du}{2x}} = \int \frac{x^2 \cdot du}{2x\sqrt{u}} = \int \frac{\sqrt{u-1} \cdot du}{2\sqrt{u}}$$

$$\cancel{= \frac{1}{2}\int \sqrt{\frac{u-1}{u}} \cdot du = \frac{1}{2}\int \sqrt{1-\frac{1}{u}} \cdot du}$$

$u = \sqrt{1+x^2}$

$du = \frac{\cancel{2}x \cdot dx}{\cancel{2}\sqrt{1+x^2}}$

$$= \int \frac{x^{\cancel{2}} \cdot \cancel{\sqrt{1+x^2}} \cdot du}{\cancel{x}\ \cancel{(u)}} = \int \sqrt{u^2-1} \cdot du$$

$du = \frac{x \cdot dx}{\sqrt{1+x^2}}$

$$= \int (u^2-1)^{\frac{1}{2}} \cdot du = \cancel{\frac{2}{3}(u^2-1)^{\frac{3}{2}}}$$

$dx = \frac{\sqrt{1+x^2}}{x} \cdot du$

$u = \sqrt{1+x^2}$

$u^2 = 1+x^2$

$x = \sqrt{u^2-1}$

(b) $\int \frac{x^2+x+1}{x^3+x} dx =$

$$\frac{x^2+x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$$

? (c) $\lim_{x\to\infty} \left(\frac{x-1}{x-2}\right)^x =$ $\lim_{x\to\infty} \frac{(x-1)^x}{(x-2)^x}$ $= \lim_{x\to\infty} \cancel{\frac{x(x(-1))^{x+1}}{x(x-2)^{x-1}}}$

$$= \lim_{u\to\infty} \frac{(u)^{u+1}}{(u)^{u+2}} = \lim_{u\to\infty} u^{(\cancel{u}+1-\cancel{u}-2)}$$

$u = x-1$

$du = dx$

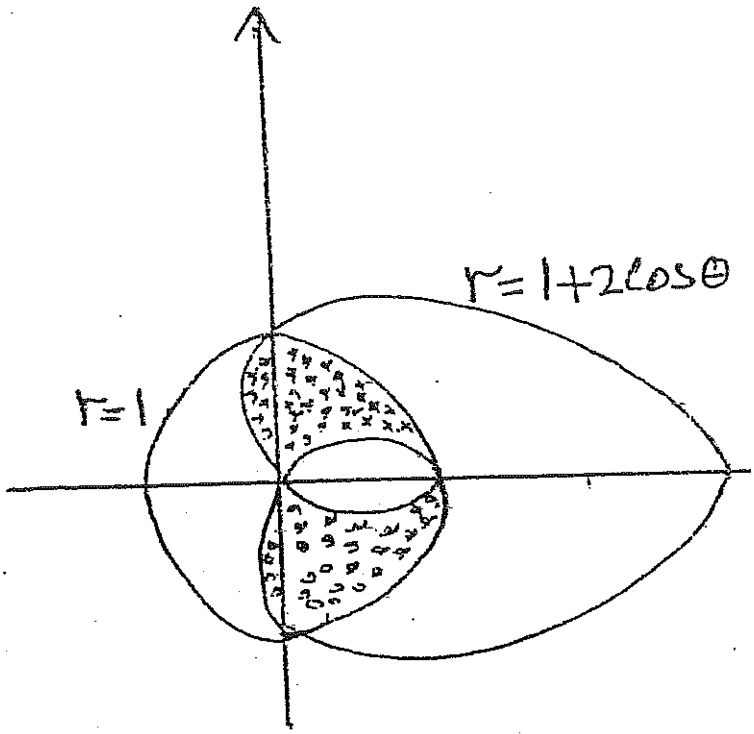
$x = u+1$

$x = \infty \quad u = \infty$

$$= \lim_{u\to\infty} u^{-1}$$

$$= \lim_{u\to\infty} \frac{1}{u} = 0$$

IV (15 points): Find the area of the shaded region.



Birzeit University
Math. & Comp. Science Dept.
Math. 132

Final Exam **Second Semester** [redacted]

The Name of Discussion Teacher:

Student Name:________________________**Number:**________ **Section**___

I (50 points) : Circle the MOST correct answer:

1. If $\sinh x = 3$ then $\cosh(-x) =$
   (a) $\sqrt{10}$
   (b) $-\sqrt{10}$
   (c) $\sqrt{8}$
   (d) $\sqrt{10}, -\sqrt{10}$

   1, 3, $\sqrt{10}$

? 2. $\int \frac{ax+b}{x^3+x^2}dx$ is a rational function if
   (b) $b = 0$
   (c) $a = b$
   (d) $a = -b$

3. If $2^{x^2} = 4.2^x$ and $x > 0$ then $x =$
   (a) 2
   (b) -1
   (c) $2, -1$
   (d) 4

   $x^2 \ln 2 = x \ln 4.2$

   $x = \frac{\ln 4.2}{\ln 2} = \ln 2.2$

4. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}dx =$
   (a) $\tanh^{-1}(1)$
   (b) $\ln(e^2+1) - \ln 2$
   (c) $\ln(e^2+1) - \ln 2 - 1$
   (d) None of the above

$u = e^x$
$du = e^x\,dx$
$x = 1 \to u = e$
$x = 0 \to u = 1$

$= \int_1^e \frac{(u - \frac{1}{u})}{u(u + \frac{1}{u})}\,du = \int_1^e \frac{\frac{u^2-1}{u}}{u\left(\frac{u^2+1}{u}\right)}\,du$

$= \int_1^e \frac{u^2-1}{u(u^2+1)}\,du = \int_1^e$

$\frac{u^2-1}{u(u^2+1)} = \frac{A}{u} + \frac{B}{u^2+1}$

$u^2 - 1 = A(u^2+1) + B(u)$

$u = 0$: $A = 1$

$u = 1$: $2A + B = 0$, $B = -2A$, $B = 2$

$= \int_1^e \left(\frac{1}{u} - \frac{2}{u^2+1}\right) \cdot du$

$= \ln u - 2\ln(u^2+1)\Big]_1^e$

$= \ln e - 2\ln(e^2+1) - \ln 1 + 2\ln(2)$

$= 1 - 2\ln(e^2+1) + 2\ln 2$

$= 2x\ln x - \frac{2(ue^u - e^u)}{\ln 3}$

$f(x) = u \quad g(x) = e^u$

$f'(x) = 1 \quad \int g(x) = e^u$

$\iint g(x) = e^u$

$= \int \frac{\ln x^2}{\ln 3} \cdot dx = \frac{1}{\ln 3} \int \ln x^2 \cdot dx$

$u = \ln x^2$

$du = \frac{2x}{x^2} \cdot dx$

$du = \frac{2}{x} \cdot dx$

$dx = \frac{x}{2} \cdot du$

$= \frac{1}{\ln 3} \int u \cdot \frac{x}{2} \cdot du$

5. $\int \log_3 x^2 dx =$

(a) $2x \log_3 x^2 - x + c$

(b) $2x \log_3 x - x + c$

(c) $\frac{2}{\ln 3}(x \ln x + x) + c$

(d) $\frac{2}{\ln 3}(x \ln x - x) + c$

$= \frac{1}{2\ln 3} \int u \cdot e^u \cdot du$

$z = e^u \quad dv = u\, du \quad x = e^{\frac{u}{2}}$

$dz = e^u \cdot du \quad v = \frac{u^2}{2} \quad \frac{u}{2} = \ln x$

? 6. If $\int_0^b f(x)dx$ diverges and $\int_a^b g(x)dx$ diverges then $\int_a^b f(x).g(x)dx$

(a) converges always

(b) diverges always

(c) Can't decide

(d) Converges if $f(x) = g(x)$

$= \frac{e^u u^2}{2} - \int \frac{u^2}{2} e^u \cdot du$

7. A line $y = ax + b$ and a curve of a function $y = e^x$ can intersect in at most

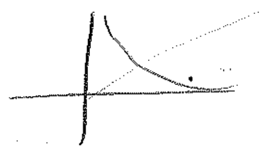
(a) 1 point

(b) 2 points

(c) 3 points

(d) 4 points

~~$ax + b = e^x$~~



8. If $\sin^{-1} x = \frac{\pi}{3}$ then $\cos^{-1} x =$

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{-\pi}{3}$

(d) π/[illegible]

$x = \frac{\sqrt{3}}{2}$

$x = \sin u$

$u = \sin^{-1} x$

$du = \frac{dx}{\sqrt{1 - x^2}}$

$dv = \sqrt{1-x^2} \cdot du$

$dx = \sqrt{1 - \sin^2 u} \cdot du$

$= \cos u \cdot du$

9. $\int_0^{\frac{\pi}{2}} \sin x\, dx + \int_0^1 \sin^{-1} x\, dx = -\cos x \Big]_0^{\pi/2} + \sin u \Big]_0^{\pi/4}$

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) $\frac{1}{2}$

$= 0 + 1 + \frac{1}{\sqrt{2}} - 0$

$\left(\frac{2+1}{\sqrt{2}}\right) \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}$

$x = 1$	$x = 0$
$u = \pi/4$	$u =$

10. One of the following is not an improper integral.

(a) $\int_0^{10} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^{a} x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int^{\infty} \frac{dx}{x^2 - 1}$

11. $\int \sin^2 x \cos^3 x \, dx =$

(a) $\dfrac{\sin^3 x \cos^4 x}{12} + c$

(b) $\dfrac{\sin^3 x}{3} - \dfrac{\sin^5 x}{5} + c$

(c) $\dfrac{\cos^3 x \sin^4 x}{12} + c$

(d) $\dfrac{\sin^3 x}{15}(5 + 3\sin^2 x) + c$

12. The function $f(x) = \ln[x + 2]$ has domain

(a) $(-2, \infty)$

(b) $(0, \infty)$

(c) $[-1, \infty)$

(d) $[0, \infty)$

13. $\int x^3 e^x \, dx =$

(a) $\frac{x^4}{4} e^x + c$

(b) $\frac{x^4}{4} e^x - e^x + c$

(c) $e^x(x^3 - 3x^2 + 6x - 6) + c$

(d) $e^x(-x^3 + 3x^2 - 6x + 6) + c$

14. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1 - [illegible]}$ [illegible] $t \le 1$, then

(a) The initial point is $(1, 0)$ and the end point is $(0, 1)$.

(b) The motion is clockwise.

(c) The motion is counter clockwise.

[illegible]

15. The total distance travelled by a particle moving on the curve $r = 2\sin\theta$ from $\theta = 0$ to $\theta = 2\pi$ is

(a) 4π

(b) 2π

(c) π

(d) 8π

16. The slope of the tangent lines to the curve $r = 1 - 2\cos\theta$ at the origin are

(a) 0

(b) $\pm\sqrt{3}$

(c) ± 1

(d) $\not$ not defined

$y = \pm \frac{b}{a} x$

$a = 4 \quad c = 5$

$b = 3$

$e = \frac{c}{a} = \frac{5}{4}$

$x = \frac{a}{e}$

17. The directrices of the hyperbola $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$ are

(a) $y = \pm\frac{16}{5}$
(b) $x = \pm\frac{16}{5}$
(c) $x = \frac{-26}{5}, x = \frac{6}{5}$
(d) $y = \frac{-26}{5}, y = \frac{6}{5}$

$D = \frac{a}{e} = \frac{4}{5} \cdot 4 = \frac{16}{5}$

$\frac{4}{\frac{5}{4}}$

18. The polar curves $r = \cos 2\theta, r = \frac{1}{2}$ intersects in

(a) 1 point
(b) 2 points
(c) 4 points
(d) 8 points

$x \ \frac{\pm 16}{5}$

$x = +\frac{16}{5} - 2 \qquad x = \frac{-16}{5} \quad -2$

19. The circles $x^2 + y^2 = 1$ and $(x-1)^2 + y = a^2$ intersect in two point if:

(a) $a = 2$
(b) $2 < a$
(c) $a < 2$
(d) $0 < a < 2$

$x \to 0$
$u \to \infty$

20. $\lim_{x\to 0^+} \frac{\ln x}{\sqrt{x}} =$

$u = \ln x$
$du = \frac{dx}{x}$

$dv = x \ln x$

$x = e^u \qquad u = \sqrt{x}$

(a) 0
(b) $-\infty$
(c) ∞
(d) 1

$= \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \frac{2\sqrt{x}}{x} = \infty$

II (15 points): 1. $\lim_{x\to\infty} \left(\frac{x-1}{x-2}\right)^x$

$?\ \frac{?}{\infty 0}$

$= \lim_{x\to\infty} \frac{(x-1)^x}{(x-2)^x} = \frac{\cancel{x}\ln(x-1)}{\cancel{x}\ln(x-2)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots}{\ln 2 - \frac{x}{2} - \frac{x^2}{4}}$

$f(x) = \ln(x-2) \quad f(0) = \ln 2$

$f'(x) = \frac{1}{x-2} \quad f'(0) = -\frac{1}{2} \qquad = \ln 2$

$f''(x) = \frac{-1}{(x-2)^2} \quad f''(0) = -\frac{1}{4}$

2. Test for convergence $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

$u = e^x$
$du = e^x \, dx$
$dx = \frac{du}{e^x} = \frac{du}{u}$

$= \int_{-\infty}^{\infty} \frac{du}{u(u+\frac{1}{u})} \quad = \int_{-\infty}^{\infty} \frac{du}{\cancel{u}(\frac{u^2+1}{\cancel{u}})}$

$= \tan^{-1} u \Big]_{-\infty}^{0} + \tan^{-1} \Big]_{0}^{\infty}$

$= \tan^{-1}(e^x) \Big]_{-\infty}^{0} + \tan^{-1}(e^x) \Big]_{0}^{\infty}$

$= \cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

? 3. Evaluate $\int \frac{\sqrt{9-x^2}}{x} dx$.

~~$u = 9 - x^2$~~
~~$du = -2x \, dx$~~
~~$dx = -\frac{du}{2x}$~~

~~$= \int \frac{\sqrt{u}}{2x^2} du$~~

~~$= \int \frac{\sqrt{u}}{2(9-u)} du$~~

~~$\int \frac{\sqrt{9-u}}{2u} du$~~

$u = x^2$
$du = 2x \, dx$
$dx = \frac{du}{2x}$

~~$a^2 - x^2$~~ $\quad x = a \sin\theta$

$a^2 - x^2 \quad x = a\sin\theta$
$dx = a\cos\theta \, d\theta$

$= \int \frac{\sqrt{a^2 - a^2\sin^2\theta}}{\cancel{a}\sin\theta} a\cos\theta \, d\theta$

$= \int \frac{\cos^2\theta}{\sin\theta} d\theta = \int \cot\theta \csc\theta \, d\theta = -\csc\theta + C$

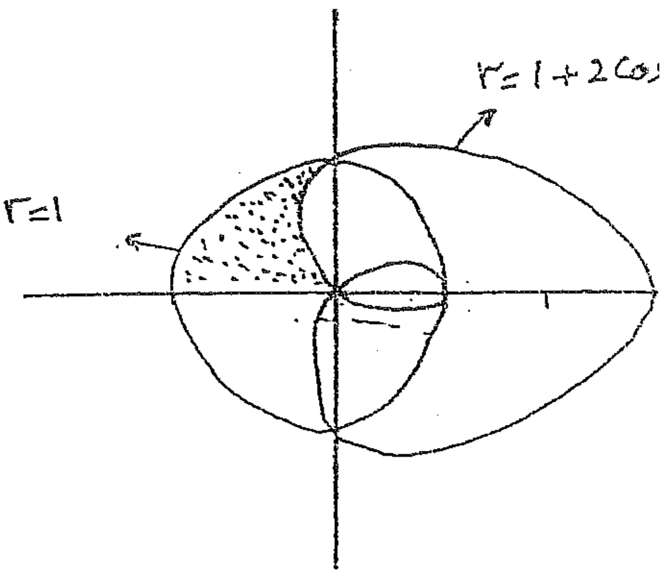
III (20 points): (a) Sketch the graph of $r = \dfrac{8}{2 - \sin\theta}$ and indicate the center, vertices, Foci and directrices.

(b) Use part (a) to sketch the graph of $r = \dfrac{8}{2 - \sin(\theta + \frac{\pi}{4})}$

IV (20 points): (a) Sketch the graph of the conic section $r = \dfrac{8}{2 - 4\cos\theta}$ indicating vertices foci and directrices in polar coordinates.

(b) Use (a) to sketch the graph of $r = \dfrac{8}{2 - 4\cos\left(\theta + \frac{\pi}{4}\right)}$ indicating vertices, foci and directrices in polar coordinates.

V (15 points): Find the shaded area.



IV (20 points): (a) Sketch the graph of the conic section $r = \dfrac{8}{2 - 4\cos\theta}$ indicating vertices foci and directrices in polar coordinates.

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70

Birzeit University- Mathematics Department
Math 132

Dr. Marwan Aloqeili(Sec.2) and Dr. $\boxed{\text{Marwan Awartani (Sec.1\&3)}}$

Fall 2002/2003

Third Exam

Name: Ali Nayef Taqatqa. Number: 1011273

There are 10 (T/F) questions and 14 multiple choice. **Calculators are not allowed.**

Question 1 Circle the most correct answer:

$\lim \frac{n}{n+10^6}$

1. $\sum_{n=1}^{\infty} \frac{1}{n+(10)^6}$
   (a) Converges to 0.
   (b) Converges by nth term test.
   (c) Diverges. ✓
   (d) None.

2. One of the following p-series converges:
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$. ✓
   (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$. $p>1$
   (c) $\sum_{n=1}^{\infty} \frac{1}{n}$.
   (d) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$.

3. The sequence $a_n = (1-\frac{1}{n^2})^n$
   (a) Diverges. ✓
   (b) Converges to -1.
   (c) Converges to 1.
   (d) None of the above.

   $\lim_{n\to\infty} (1-\frac{1}{n^2})^n$
   $= n\ln(1-\frac{1}{n^2})$
   $= \ln(1-\frac{1}{n^2}) + \ldots$
   $= 0 + 0 = 0$
   $e^0 = 1$

4. $\sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1} n =$
   (a) $\pi/2$.
   (b) $\pi/4$. ✓
   (c) $-\pi/4$.
   (d) None of the above.

   $\tan^{-1}(2) - \tan^{-1}(1) + \cancel{\tan^{-1}(3)} - \cancel{\tan^{-1}(2)} + \tan^{-1}(4) - \cancel{\tan^{-1}(3)}$

5. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$
   (a) Diverges by integral test.
   (b) Diverging by directly comparing it with $\sum \frac{1}{n}$. ✓
   (c) Diverging by limit comparison test with $\sum \frac{1}{n}$.
   (d) All of the above.

$\frac{1}{2}\ln^2 b - \frac{1}{2}\ln^2 2$

$\frac{\ln n}{n} > \frac{1}{n}$

$\lim \frac{n \ln n}{n} = \infty$

$\int \frac{\ln n}{n} = \int \frac{u}{x} x\,du = \frac{u^2}{2}\Big|_1^b$

$\lim_{n\to\infty} \left(1 - \frac{1}{n^2}\right)^n$

$\ln f(x) = \frac{\ln\left(1-\frac{1}{n^2}\right)}{1/n}$

$= \frac{\frac{1}{1-1/n^2} \cdot \frac{2}{n^3}}{-1/n^2}$

$= \lim \frac{-1}{(1-\frac{1}{n^2})n^3}$

$= \lim \frac{-1/n}{1-\frac{1}{n^2}}$

$\lim \ln f(x) = 0$

$\Rightarrow \ln f(x) = 1$

$\Rightarrow \lim f(x) = e^0$

$(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \ldots$
$+ (\tan^{-1} 4 - \tan^{-1} 3) + \ldots + (\tan^{-1} n$
$- \tan^{-1} n)$ $\quad \frac{\pi}{2} - \frac{\pi}{4}$

$S_n = -\tan^{-1} 1 + \tan^{-1}(n+1) = \frac{\pi}{4}$

$\frac{1+\cos^2 n}{n^2} < \frac{2}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{1+(\cos n)^2}{n^2}$

(a) Converges.

(b) Diverges.

(c) Diverges by the nth term test.

(d) Cannot determine.

7. $\sum_{n=1}^{\infty} e^{-n} = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}$ $\quad \left(\frac{1}{e}\right)^n$

(a) $\frac{e}{e-1}$.

(b) $\frac{1}{e-1}$.

(c) $\frac{1}{1-e}$.

(d) None of the above.

$\frac{1}{e^n} = \left(\frac{1}{e}\right)^n$ $\quad = \frac{1}{1-\frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$ $\quad a = \frac{1}{e} \quad r = \frac{1}{e}$

$\frac{1/\cancel{e}}{e(1-\frac{1}{e})} = \frac{1}{e-1}$

8. One of the following is true:

(a) $\sum \frac{3^n}{n^3 2^n}$ diverges by ratio test.

(b) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ converges by nth root test. ×

(c) $\sum_{n=1}^{\infty} 2^n$ converges. ×

(d) $\sum \frac{1}{\ln n}$ is a geometric series. ×

$\lim \frac{3^{n+1}}{(n+1)^3 2^{n+1}} \cdot \frac{n^3 2^n}{3^n}$

$= \lim \frac{n^3}{(n+1)^3}$

$= \frac{3}{2}$

9. The nth partial sum of the series $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is

(a) $s_n = 1 - \frac{1}{\sqrt{2}}$.

(b) $s_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$.

(c) $s_n = 1 - \frac{1}{\sqrt{n+1}}$.

(d) None of the above.

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$ $\quad 1 - \frac{1}{\infty} = 1$

$\left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

$\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}\right) + \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}}\right)$

$\dots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

$= 1 - \frac{1}{\sqrt{n+1}}$

10. If $a_1 = 2$, $a_{n+1} = \frac{2}{n}a_n$ then $\sum_{n=2}^{\infty} a_n$

(a) Converges by ratio test.

(b) Diverges.

(c) Converges by integral test.

(d) None of the above.

$2 + 2(2) + 1(4) + ($

$\lim \frac{a_{n+1}}{a_n}$

$= \frac{\frac{2}{n} \cancel{a_n}}{\cancel{a_n}} = \frac{2}{n} = 0$

11. The series $\sum_{n=0}^{\infty} (\ln x)^n$ converges if

(a) $-1 < x < 1$.

(b) $-e < x < e$. ×

(c) $e^{-1} < x < e$.

(d) None.

$\sum_{n=1}^{\infty} (\ln x)^{n-1}$

$a_1 = (\ln x)^0 = 1$

$a_2 = (\ln x)$

$a_3 = (\ln x)^2$ $\quad \Rightarrow r = (\ln x)$

$S_n = \frac{1}{1-(\ln x)}$

$e^{-1} < e^{\ln x} < e^1$

12. One of the following series converges:

(a) $\sum_{n=1}^{\infty} n^2$.

(b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(d) $\sum_{n=1}^{\infty} \ln(1/n)$.

$-5 \sum$

13. $\sum_{n=1}^{\infty} \frac{-5}{n}$

(a) Converges to 0.

(b) Converges to 1.

(c) Diverges.

(d) None of the above.

14. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(a) Converges.

(b) Diverges.

(c) Converges to 0.

(d) None.

$\lim \frac{2}{(n+1)^2} \cdot \frac{n^2}{\cancel{2}}$

$= 2\lim \frac{n^2}{n^2+2n+1}$

$= 2 > 1 \Rightarrow div.$

Question 2. Answer by True or False:

30

1. The series $\sum_{n=1}^{\infty}(-1)^{n+1}$ diverges. (true)
2. If $\lim_{n\to\infty}(a_n)^{1/n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges. (false)
3. Any increasing sequence and bounded from above converges. (true)
4. $\lim_{n\to\infty} \frac{x^n}{n!} = 0$. (true)
5. The sequence $a_n = 1 + (-1)^n$ diverges. (true)
6. If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges. (false)
7. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is called the harmonic series. (false)
8. The sequence $a_n = \frac{3n+1}{n+1}$ is nondecreasing. (true)
9. The series $\sum_{n=1}^{\infty}(\sin x)^n$ converges for any value of x. (false)
10. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} ka_n$, k is any constant, converges. (true)

$\lim_{n\to\infty} \frac{(-1)^{n+2}}{(-1)^{n+1}} = -1$

0, 1, 0, 1

$\sum_{n=2} (-1)^n$

$\frac{3(n+1) - (3n+1)}{(n+1)^2}$

$\frac{3n+3-\dots}{(n+1)^2}$

$= \frac{2}{\dots}$

Birzeit University

Math. & Comp. Science Dept.

Math. 132

Third Hour Exam **Second Semester** [redacted]

The Name of Discussion Teacher:

Student Name:______________________________**Number:**__________ **Section**____

I (40 points): Circle the MOST correct answer:

1. The directrix of the parabola $y = 2x^2 + 4x + 1$ is
   (a) $y = -\frac{1}{8}$
   (b) $y = \frac{7}{8}$
   (c) $y = -\frac{9}{8}$
   (d) $y = \frac{9}{8}$

2. A circle of radius a and an ellipse of major semiaxis a both centered at the origin meets in
   (a) 1 points
   (b) 1 points
   (c) 2 point
   (d) 0 point

3. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents
   (a) Parabola
   (b) Ellipse
   (c) Hyperbola
   (d) Circle

4. A parametrization of the line segment with initial point $(0,1)$ and terminal point $(1,0)$ is
   (a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \le t \le \frac{\pi}{2}$
   (b) $x = 1 - t$, $y = t$, $0 \le t \le 1$
   (c) $x = t$, $y = 1 + t$, $-1 \le t \le 0$
   (d) $x = 1 + t$, $y = t$, $-1 \le t \le 1$

5. The Parametrization $x(t) = -\cos t$, $y(t) = \sin t$, $0 \leq t \leq 2\pi$

(a) A circle with initial point $(1,0)$ counterclockwise.
(b) A circle with initial point $(-1,0)$ counterclockwise.
(c) A circle with initial point $(-1,0)$ clockwise.
(d) A circle with initial point $(1,0)$ clockwise.

6. The length of the curve $x(t) = \sin t$, $y(t) = 1 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$ is:

(a) π
(b) 2π
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$

7. The slope of the tangent line to the curve $y(t) = t^2 - \sin t$, $x(t) = \cos t$, at $t = \frac{\pi}{2}$ is:

(a) $\frac{\pi^2}{2}$
(b) $\frac{-1}{\pi}$
(c) -2
(d) $-\pi$

8. Which one of the following points lies on the curve $r = \cos 2\theta$

(a) $(0,0)$
(b) $(1, \frac{\pi}{2})$
(c) $(\frac{1}{2}, \frac{\pi}{3})$
(d) all

9. The graph of $r = 2\csc\theta$ is

(a) Circle

(c) Hyperbola
(d) Parabola

10. The curves $\theta = \frac{\pi}{2}$ and $r = 0$

(a) never meets.
(b) intersect in one point.
(c) intersect in infinitely many points.
(d) are identical.

11. $7^{\log_7^5} =$

(a) $\frac{5}{7}$
(b) $\frac{7}{5}$
(c) 5
(d) 7

12. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

(a) $y = -\dfrac{1}{8}$
(b) $y = \dfrac{7}{8}$
(c) $y = -\dfrac{9}{8}$
(d) $y = \dfrac{9}{8}$

13. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

(a) Parabola
(b) Ellipse
(c) Hyperbola
(d) Circle

14. A parametrization of the line segment with initial point (0,1) and terminal point (1,0) is

(a) $x = \sin^2 t$; $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$
(b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$
(c) $x = 1$, $y = 1 + t$, $-1 \leq t \leq 0$
(d) $x = 1 + t$, $y = 1$, $0 \leq t \leq 1$

15. $\dfrac{\log_2(x)}{\log_8(x)} =$

(a) $\dfrac{2}{8}$
(b) $\dfrac{1}{3}$
(c) $\log_4^x$
(d) 3

16. $\sec^{-1}(-2)$

(a) $\dfrac{2\pi}{3}$
(b) $-\dfrac{\pi}{3}$
(c) $-\dfrac{2\pi}{3}$
(d) $\dfrac{3\pi}{\ }$ [illegible]

17. $\lim\limits_{x \to 0^+} (\ln x - \ln(\sin x))$

(a) 0
(b) -1
(c) $+1$
(d) Doesn't exist.

18. $\lim\limits_{x \to \infty} (\ln x)^{\frac{1}{x}}$

(a) ∞
(b) 0
(c) 1
(d) Doesn't exist

19. $\lim\limits_{x \to \frac{\pi^-}{2}} \dfrac{\sec x}{\tan x} =$

(a) 0
(b) 1
(c) ∞
(d) Doesn't exist

20. The order of the functions $x^2, e^x, \ln x$ from slowest growing to fasters growing as $x \to 0$ is

(a) e^x, $\ln x$, [illegible]
(b) e^x, x^2, $\ln x$
(c) x^2, e^x, $\ln x$
(d) $\ln x$, x^2, e^x

II (20 points): Suppose the path of a moving particle in a plane is described by

$$\begin{matrix} x(t) = 3 + 4\sin(t) \\ y(t) = 2 + 5\cos(t) \end{matrix}, \quad 0 \le t \le \pi$$

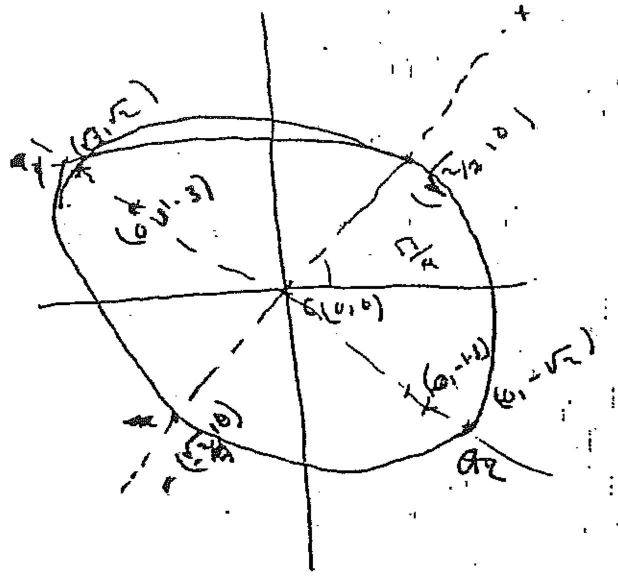
1. Sketch the path of motion and determine the direction.
2. Find the equation of the tangent line at $t = \frac{\pi}{3}$.

Question #4:

Graph the conic section $2x^2 + 3xy + 2y^2 - 1 = 0$ in the xy-plane indicating the center, the vertices and the foci in the xy coordinates.

$$2x^2 + 3xy + 2y^2 - 1 = 0$$

$B^2 - 4AC = 9 - 4(2)(2) < 0$ ellips



$$\cot 2\alpha = \frac{A-C}{B} = \frac{2-2}{3} = 0$$

$$2\alpha = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\cos\frac{\pi}{4}$$

$$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

Equation became

$$2\left(\frac{x'-y'}{\sqrt{2}}\right)^2 + 3\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) + 2\left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 1 = 0$$

$$\cancel{2}\frac{(x'^2 + y'^2 - 2x'y')}{\cancel{2}} + 3\left(\frac{x'^2 - y'^2}{2}\right) + \cancel{2}\left(\frac{x'^2 + y'^2 + 2x'y'}{\cancel{2}}\right) - 1 = 0$$

$$x'^2 + y'^2 + \frac{3x'^2}{2} - \frac{3y'^2}{2} + x'^2 + y'^2 = 1$$

$$\frac{7}{2}x'^2 + \frac{1}{2}y' = 1 \qquad \frac{x'^2}{2/7} + \frac{y'}{2} = 1$$

$$c = \sqrt{a^2 - b^2} = \sqrt{2 - \frac{2}{7}} = \sqrt{1.714} = 1.3$$

Foci

$$c = \sqrt{\frac{192}{49}} = 1.98 \approx 2$$

Foci at $x'y'$ plan $= (0, \mp 1.3)$

at xy plan $\quad y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$

$$= 0 + 1.3(\cos \pi/4)$$

$$= \frac{1.3}{\sqrt{2}}$$

$$x = x'\cos\frac{\pi}{4} - y'\cos\frac{\pi}{4}$$

Foci in xy plan $=$

Foci $\boxed{\left(-\frac{1.3}{\sqrt{2}}, -\frac{1.3}{\sqrt{2}}\right) \quad \left(\frac{1.3}{\sqrt{2}}, \frac{1.3}{\sqrt{2}}\right)}$

vertics $\quad a = (0, \mp\sqrt{2})$

$x = \cancel{x'}\cancel{\cos}x - y'\sin x$

$x = 0 - \frac{\sqrt{2}}{\sqrt{2}} = -1$

$\boxed{3}$ ~~V~~ertics in xy plang

$a_1 = (-1, 1)$

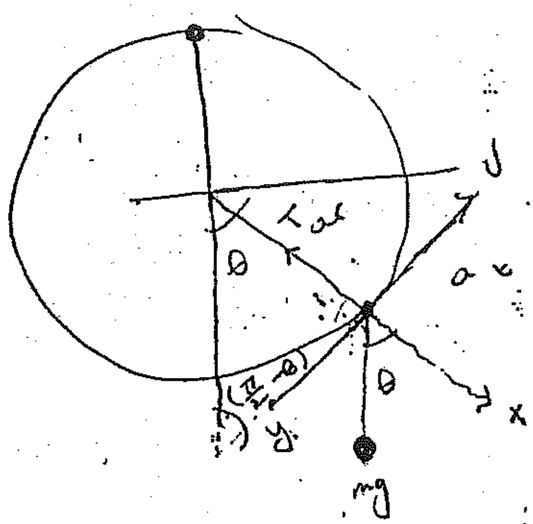
$a_2 = (1, -1)$

$y = x'\cancel{\sin x} + y'\cos x$

$= \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\frac{mv^2}{R} = mg \quad T = 0$

$\boxed{v^2 = \sqrt{Rg}}$



$\frac{mv^2}{R}$

$\boxed{mg\cos\theta + \frac{mv^2}{R} = T}$ — ①

$\boxed{a_T = -mg\sin\theta}$

Q#7: Find the series radius of convergence. For what values of x does the series converge :

a) Conditionally

b) absolutely

(i) $\dfrac{(x-2)^n}{n}$ (ii) $\dfrac{(-1)^n x^n}{n!}$

Q#8: a) Find the maclurin series for $f(x) = \ln(1+x^2)$.

b) How many terms are supposed to be used to get an estimate f(0.2) with error less that 10^{-6}.

#9: **Evaluate:** $\sum_{n=1}^{\infty} nx^n$ if $|x| < 1$

MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min. **JANUARY 2008**

NAME: George Hannoneh

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Jad'on

82

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]

CIRCLE THE RIGHT ANSWER:

28
30
10
14
82

1. Let $f(x)=\sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x, $\int_0^x f(t)dt$ is

$\frac{x^{n+1}}{n+1}\Big|_0 = \frac{x^{x+1}}{x+1} - x$?

$\frac{x^{x+1} - x(x+1)}{x+1}$

$\ln|1+x|$

(A) $x = 0$ only
(B) $|x| \le 1$
(C) $-\infty < x < \infty$
(D) $-1 \le x < 1$
(E) $-1 < x < 1$

$-\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16}$

2. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

(A) -1/24
(B) 1/24
(C) 1/96
(D) -1/384
(E) 1/384

$\frac{(-1)^n (x)^n}{2^{n+1} \, n!}$ $\frac{1}{32} \frac{x^4}{4\times3\times2}$

$\frac{1}{e^{\frac{x}{2}}}$ $-\frac{e^{-\frac{x}{2}}}{2}$ $\frac{e^{-\frac{x}{2}}}{4}$ $-\frac{1}{2e^{\frac{x}{2}}}$ $\frac{1}{4e^{\frac{x}{2}}}$

3. Which of the following series diverges?

(A) $\Sigma 1/n^2$ conv
(B) $\Sigma 1/(n^2 + n)$ conv.
(C) $\Sigma n/(n^3 + 1)$
(D) $\sum \frac{n}{\sqrt{(4n^2-1)}}$
(E) none of the preceding.

$\frac{n}{n\sqrt{4-\frac{1}{n^2}}}$ $\frac{1}{4}$

4. For which of the following series does the Ratio Test fail?
   (A) $\Sigma 1/n!$
   (B) $\Sigma n/2^n$
   (C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \ldots$
   (D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \ldots$
   (E) $\Sigma n^n/n!$

5. Which of the following alternating series diverges?
   (A) $\Sigma (-1)^{n-1}/n$
   (B) $\Sigma (-1)^{n+1}(n-1)/(n+1)$
   (C) $\Sigma (-1)^{n+1}/\ln(n+1)$
   (D) $\sum \dfrac{(-1)^{n-1}}{\sqrt{n}}$
   (E) $\Sigma (-1)^{n-1} n/n^2 + 1$

6. Which of the following series converges conditionally?
   (A) $3 - 1 + 1/9 - 1/27 + \ldots$
   (B) $\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} - \ldots\ldots\ldots\ldots$
   (C) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$
   (D) $1 - 1.1 + 1.21 - 1.332 + \ldots$
   (E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \ldots$

7. Suppose *f(x)* is a function with Taylor series converging to *f(x)* for all $x \in \Re$.

   If $f(0) = 2, \quad f'(0) = 2$ *and* $f^{n}(0) = 3$ for $n \geq 2$ then *f(x)*=

   (A) $3e^{x} + 2x - 1$ 2 5

   (B) $e^{3x} + 2x + 1$ 2

   (C) $e^{3x} - x + 1$ 2

   (D) $3e^{x} - x - 1$ 2 0 2

   (E) $3e^{x} + 5x + 5$ 4

8. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(A) I only (B) II only (C) III only

(D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

(A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!\,e}$

(D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

$\frac{x^n}{n!}$

$\frac{e(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{1}{2} \quad \frac{a_{n+1}}{a_n} \le \frac{n+4}{2n+1} \quad \text{for all } n. \text{ Then } \lim_{n\to\infty} a_n =$$

(A) 0 (B) 1/2 (C) 1

(D) 2 (E) 4

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \le \frac{1}{2}$

$\Rightarrow \sum a_n$ converges

$\Rightarrow \lim_{n\to\infty} a_n = 0$.

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ $\Rightarrow$ Converges by integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99}\Big|_3^{\infty}$$

$$= \frac{-1}{99(\ln n)^{99}}\Big|_3^{\infty} = \frac{-1}{\infty} + \frac{1}{99(\ln 3)^{99}} \Rightarrow \text{Integral Converges}$$

(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$ ⇒ Converges by the nth Root test absol

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n\to\infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

(c) $\sum_{n=1}^{\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3}$ diverges by limit comparison test

$$\lim_{n\to\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3} \div \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{n^4-n^{\frac{3}{2}}+6n}{n^4-n-3}$$

$= 1 \Rightarrow$ ~~bot~~ both diverge or converg

$\frac{1}{n}$ diverges. (power series with $p=1$)

⇒ both diverg

30

QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$

(a) When $x = -4$ does this series converge or diverge?
(b) Determine all values for which the series converges.

$$\lim_{n\to\infty} \sqrt[n]{\frac{(-2)^n}{\cancel{n^{\frac{1}{2}}}}(x+3)^n}$$

$$= \frac{-2}{n^{\frac{1}{2n}}}(x+3) \quad \therefore \quad |x+3| \le \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-\frac{1}{2} < x+3 < \frac{1}{2}$$
$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n(-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n\to\infty} 2^n = \infty$$

both diverg by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

$\Rightarrow$ the series converges on the interval $(-4, -2]$

10

QUESTION FOUR: [16 points]

Consider the integral $\int x\cos(x^3)dx$.

(a) Write down the Maclaurin series for cos(x), $\cos(x^3)$, and x $\cos(x^3)$.

(b) Evaluate $\int_0^1 x\cos(x^3)dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{3n!}$$

Maclaurin

$$\cos(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!}$$

$$x\cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x\cos(x^3) = x + \frac{x^7}{2!} + \cancel{\frac{x^{19}}{4!}} + \cancel{\frac{x^{19}}{6!}} + \cancel{\frac{x^{25}}{8!}} + \cancel{R}$$

$$\int_0^1 x\cos(x^3) = \frac{2x + x^7}{2}\Big|_0^1 \quad \cancel{\text{where } R \le a_5}$$

$$\int_0^1 x\cos(x^3) = \boxed{\cancel{3/2}}$$

14

$\frac{\pi}{4} \times \frac{2\pi}{4}$, $\frac{3\pi}{4}$ $\frac{\pi}{4} \times \frac{4\pi}{4}$ $\frac{5\pi}{4}$ $\frac{\pi}{4} \times \frac{6\pi}{4}$

Key

Birzeit University
Department of Mathematics
Math 132

Final exam
Name : ....
Instructor:............

Summer/2009
Number:...
Section :......

Q#1 (72%) Circle the correct answer.

$\ln \frac{\pi}{3} \times \frac{6}{\pi}$

1) $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1+x^2)\tan^{-1} x} =$

a) $\ln 4 - \ln 3$

b) $\ln 2$

c) $\ln 3 - \ln 2$

d) $\frac{\pi}{12}$

2) If $y = (\ln x)^x$ then $\frac{dy}{dx}$

a) $(\ln x)^x \left(\frac{1}{\ln x} + \ln x\right)$

b) $(\ln x)^{x-1}$

c) $x(\ln x)^{x-1}$

d) $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$

3) $\displaystyle\int_1^{e^3} \frac{1}{x\sqrt{(1+\ln x)}} dx =$

a) $\ln \frac{4}{3}$

b) 2

c) $\frac{4}{3}$

d) $\ln \frac{3}{2}$

4) $\displaystyle\int_1^2 \frac{\sinh(\ln x)}{x} dx =$

a) 0

b) 1

c) $\frac{1}{4}$

d) $\frac{1}{2}$

5) if $y=\tan^{-1}(\frac{1}{x})$ then csc y =

a) $\frac{\sqrt{1+x^2}}{x}$ b) $\sqrt{1+x^2}$ c) $\frac{x}{\sqrt{1+x^2}}$ d) $\frac{1}{\sqrt{1+x^2}}$

6) if $y=5^{\ln x}$ then $\frac{dy}{dx}$ when x=1 is:

a) 0 b) $-\ln 5$ c) $\ln 5$ d) 1

7) $\int_0^1 e^{\sqrt{x}}\, dx =$

a) 0 b) 4 c) 3 d) 2

8) $\int x^2 e^{3x}\, dx =$

a) $\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c$

b) $\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}-\frac{2}{27}e^{3x}+c$

c) $\frac{1}{3}x^2e^{3x}-\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c$

d) $-\frac{1}{3}x^2e^{3x}+\frac{2}{9}xe^{3x}-\frac{2}{27}e^{3x}+c$

9) $\int \frac{dx}{(x+2)\sqrt{x^2+4x}} =$

a) $\sqrt{x^2+4x}+c$

b) $\frac{1}{2}\sec^{-1}\left|\frac{x+2}{2}\right|+c$

c) $\frac{1}{2}\ln\left|x^2+2x\right|+c$

d) $\sinh^{-1}(x+2)+c$

10) If $f(x) = xe^x$, then $(f^{-1})'(e) =$

(a) $\frac{1}{e^e(e+1)}$ (b) $e^e + ee^e$ (c) $\frac{1}{2e}$ (d)None of the above

$xe^x + e^x$

11) Consider the improper integrals:

(i) $\int_3^\infty \frac{dx}{(x-3)^2}$ $\left.\frac{-1}{x-3}\right|_3$ (ii) $\int_{-5}^\infty \frac{dx}{\sqrt{x+5}}$

(a) only integral (i) converge

b)only integral (ii) converges

(b) both integrals converge

d)both integrals diverge.

12. The power series $x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n} + \cdots$ converges if and only if

$\frac{x}{1-x}$ $1 + x + x^2 + \cdots + x^{n-1}$

(a) $-1 < x < 1$

(b) $-1 \le x \le 1$

(c) $-1 \le x < 1$

(d) $-1 < x \le 1$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

13. The Maclaurin series of order 3 for $f(x) = \sqrt{x+1}$ is

(a) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$

(b) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(c) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$

(d) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

??

14) Determine whether $\int_2^{30} \frac{dx}{(x-3)^{2/3}}$ converges or diverges. If the integral converges, find its value

a)converges, 15

b)converges, 3

c)converges, 1

d)diverges.

$\int_2^3 + \int_3^{30}$

$\left. 3\sqrt[3]{x-3} \right|_2^3 + \left. 3\sqrt[3]{x-3} \right|_3^{30}$

$0 - 3\sqrt{-1}$ $9 - 0$

3 3

15) If $(\frac{1+i}{1-i})^4 + z = 2+i$ then z=

a) 2-i

b) 1-2i

c) 2+4i

d) 1+i

16) The series $\sum_{1}^{\infty} \frac{n^n}{2^n 3^n}$

a) Diverges by Ratio test

b) Converges by n^{th} term test

c) Converges by Integral test

d) Converges by n^{th} root test

17) The series $\sum_{1}^{\infty} (-1)^n (\frac{n^2}{2n^4+5})$

a) Converges by limit comparison test

b) Diverges by n^{th} term test

c) Converges by n^{th} term test

d) Converges absolutely

18) The sequence $\{a_n\} = \{\frac{1+(-1)^n}{n}\}$

a) Converges to 1

b) Converges to 0

c) Converges to 2

d) Diverges

19) $\int \sin^{-1} x \, dx =$

a) $x \sin^{-1} x - 2\sqrt{1-x^2} + c$

b) $x \sin^{-1} x + 2\sqrt{1-x^2} + c$

c) $x \sin^{-1} x - \sqrt{1-x^2} + c$

d) $x \sin^{-1} x + \sqrt{1-x^2} + c$

$u = \sin^{-1} x$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} dx$ $\quad v = x$

$x \sin^{-1} x + \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

20) $\int \frac{dx}{\sqrt{2x - x^2}} =$

a) $2\sqrt{2x - x^2} + c$

b) $\sin^{-1}(x-1) + c$

c) $\sin^{-1}(x-2) + c$

d) $\sec^{-1}(x-1) + c$

21) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{(n+1)(n+2)}}$

a) Converge conditionally

b) Converge absolutely

c) Diverges

d) Converges to 2

22) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\frac{1}{\ln n} > \frac{1}{n}$ ✓

a) Converges Absolutely

b) Converges conditionally

c) Divergess by alternating series theorem

d) Diverges by nth-term test

$e^x = e^1$

23) If cosh x + sinhx=e then x=

a) e

b) 1

c) $\ln 5$

d) $\ln \sqrt{5}$

24) $\int_3^4 \frac{3dx}{x^2 + x - 2} =$

a) $\ln\frac{5}{4}$

b) $\ln\frac{4}{5}$

c) $\ln\frac{8}{5}$

d) $\ln\frac{5}{8}$

$\sum \frac{1}{x-1} - \frac{1}{x+2}$

$\ln \frac{x-1}{x+2} \Big|_3^4 = \ln \frac{\cancel{3}}{\cancel{6}_2} \times \frac{5}{2}$

Q2(9%) Use series to find an estimate for $\int_0^{\frac{1}{2}} \frac{\tan^{-1}x}{x} dx$ with an error of magnitude less than 10^{-3}

Q3(10%) Solve $z^4 = -81$ in the field of complex numbers

Q4)(10%) Sketch the graph of the conic section

$$9x^{2}+25y^{2}+18x-100y=116$$ indicating

a)foci ....

b) vertices ....

c)eccentricity...

d) directrices..